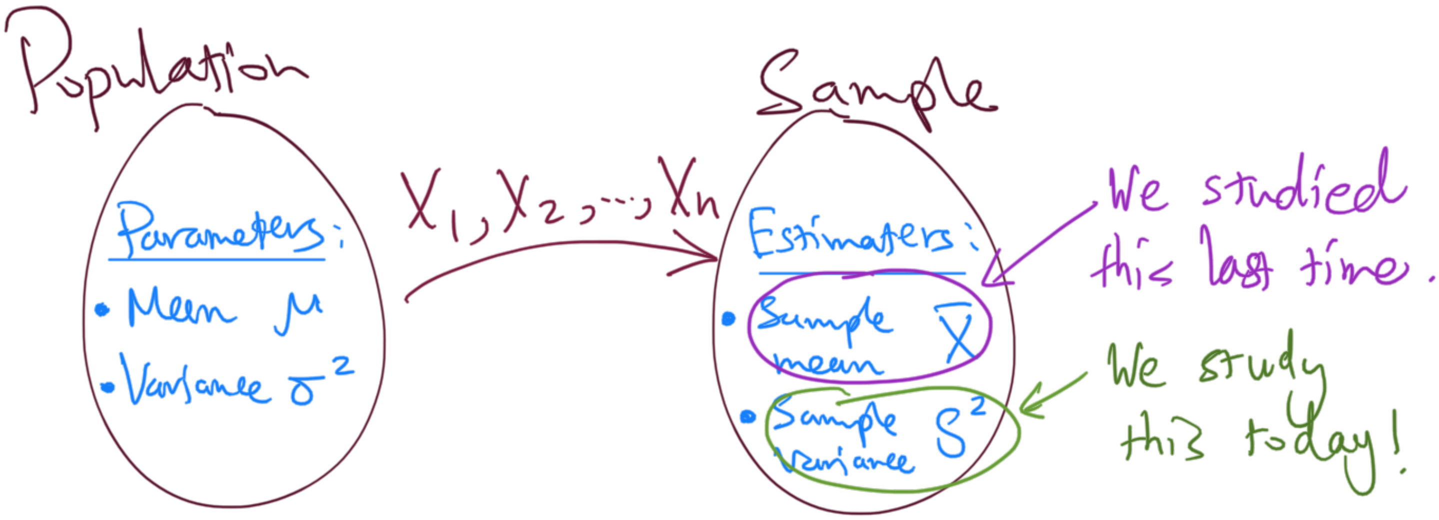
The distribution of the sample variance.



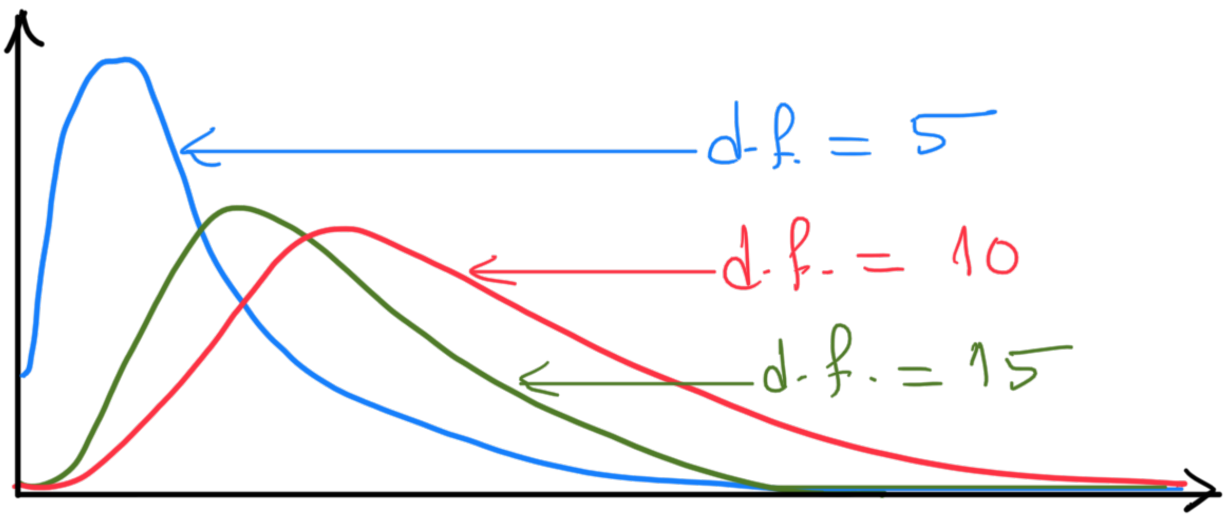
Assume that $X_1, X_2, \dots, X_n$ is a random sample (r.s.) drawn from a normal population with mean μ and variance σ^2. Let S^2 be the sample variance then $\chi^2 = \frac{(n-1)S^2}{\sigma^2}$ has a sampling distribution called the <u>chi-squared distribution</u> with $(n-1)$ degrees of fredom (denoted by χ^2-dist. with $(n-1)$ d.f.'s).

We use the following notation:-

If $X_1, X_2, \dots, X_n \overset{\text{r.s.}}{\sim} N(\mu, \sigma^2)$,

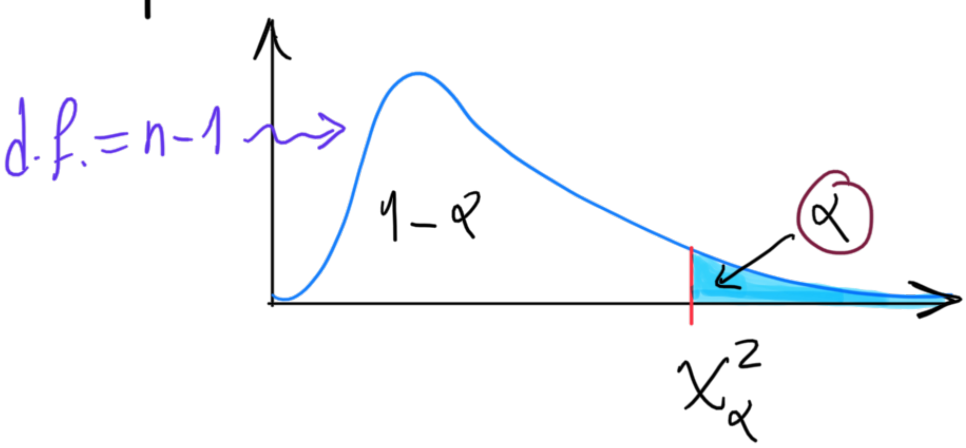
then $\chi^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

Chi-squared with d.f. $= n-1$.



Note that the curve of χ^2-distribution is skewed to the right and the shape varies with the sample size n, or percisely, the d.f.s associated with S^2.

Chi-squared distribution tables.



d.f.	$\chi^2_{0.100}$	$\chi^2_{0.050}$	$\chi^2_{0.025}$	$\chi^2_{0.010}$	$\chi^2_{0.005}$
1	2.70554	*	*	*	*
2	4.60517	*	*	*	*
3	*	*	*	*	*
4	*	*	*	*	*
5	9.23635	*	*	*	*
6	*	*	*	*	*
7	*	*	*	*	*
8	*	*	*	*	*
9	14.6837	*	*	*	*
10	*	*	*	*	*

What does this mean?

It means that, when d.f. = 5, we have

$$P(\chi^2 > 9.23635) = 0.10 \leftarrow \alpha$$

Ex. If a sample of size $n = 6$ is drawn from a population with variance $\sigma^2 = 10$, find $P(S^2 > 18.4727)$.

Soln. $n = 6$, so d.f. $= n-1 = 6-1 = 5$.

$$P(S^2 > 18.4727) = P\left(\frac{(n-1)S^2}{\sigma^2} > \frac{(n-1)(18.4727)}{\sigma^2}\right)$$

$$= P\left(\chi^2 > \frac{(5)(18.4727)}{10}\right)$$

$$= P(\chi^2 > 9.2364)$$

$$= 0.10.$$

Ex. Let $X_1, X_2, \ldots, X_{10} \sim N(\mu, 25)$. $\leftarrow \sigma^2$ If S^2 is the sample variance, find the 90^{th} percentile of S^2.

Soln. Want:- P_{90}. We know that $P(S^2 < P_{90}) = 0.90$.

Then $P\left(\chi^2 < \frac{(n-1)P_{90}}{\sigma^2}\right) = 0.90$.

$P\left(\chi^2 < \frac{(10-1)P_{90}}{25}\right) = 0.90$. (Here, d.f. $= 9$).

From χ^2-dist. table, we have $\frac{9P_{90}}{25} = 14.6437$.

Thus, $P_{90} = \frac{25}{9} * 14.6437 = 40.7881$.

<u>Ex</u>. Let $X \sim \chi^2(10)$. Find;

a) the 10^{th}-percentile of X.

b) the 95^{th}-percentile of X.

c) the 99^{th}-percentile of X.

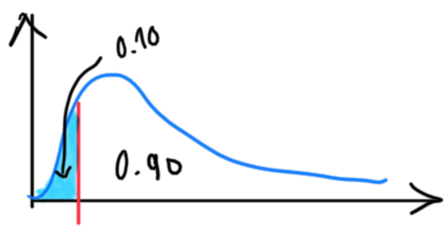
Ready!

(No need to convert to χ^2).

<u>Soln</u>. d.f. $= 10$.

a) Want P_{10}. We know that $P(X < P_{10}) = 0.10$.

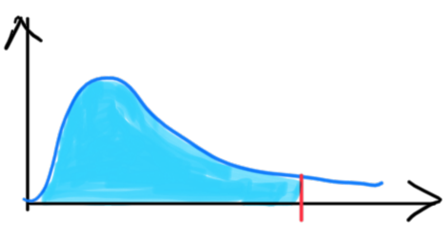
From chi-squared tables, we have $P_{10} = 4.87$.



$P_{10} = \chi^2_{0.90}$
$= 4.865180$

b) Want P_{95}. We know that $P(X < P_{95}) = 0.95$.

From chi-squared tables, we have $P_{95} = 18.31$.



$P_{95} = \chi^2_{0.05}$
$= 18.3070$

c) Exc. <u>Final Ans</u>. $P_{99} = 23.21$.

Searching keywords:

- Sampling distributions, distribution of the sample variance
- Chi-squared distribution and Chi-squared tables
- Find the probability of
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx

For any comments or concerns, please use my email to contact me.

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