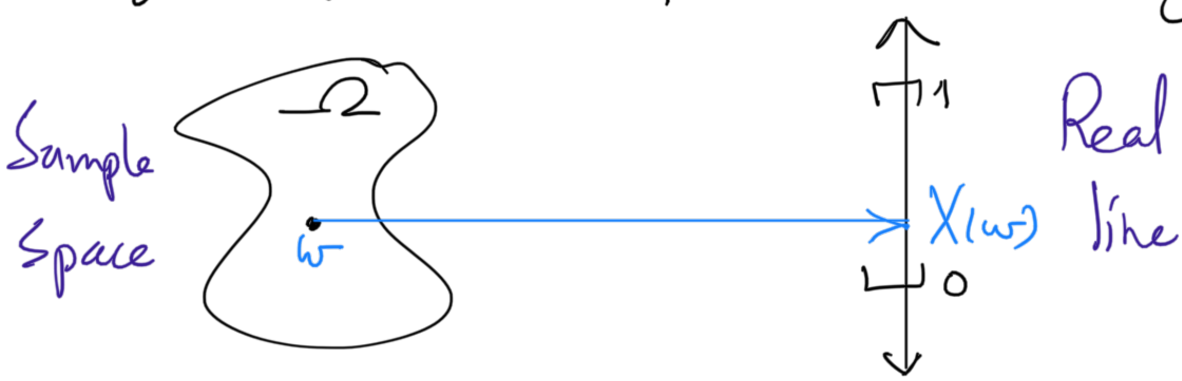
Univariate Random Variables

Def: A random variable (r.v.) X is a real-valued func. of the outcomes of a sample space Ω, that assigns to each outcome $w \in \Omega$ a real number $X(w) = x$ in the range (space or support) of X, say R_X.



the set of all possible values that occurs for X.

Random Variable

Discrete

if r.v. takes values

$R_X = \{X : X = x_1, x_2, \ldots, x_n\}$ finite

or $R_X = \{X : X = x_1, x_2, \ldots\}$ infinite but countable

Continuous

if the range of the r.v. (R_X) is an interval or collection of intervals.

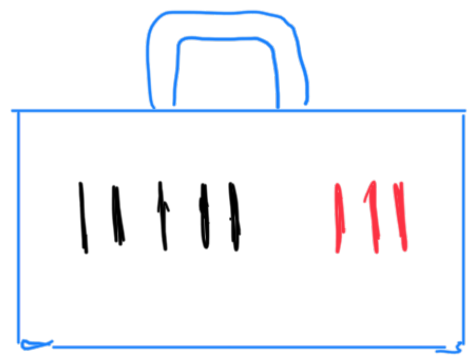
Fact: With each $x_i \in R_X$, we associate a number

$$P(X=x_i) = P_X(x_i) = P(x_i) = f(x_i)$$

which is called the probability of x_i

Def. The func. f defined above is called the probability density func. (p.d.f.) of the r.v. X.

Ex. A bag contains 5 black pens and 3 red pens. 3 pens are taken from this bag.



Let X be the number of black pens within the taken ones.

(1) What are the possible values of X.

Soln. $X=0$, $X=1$, $X=2$, $X=3$.

(2) Find $P(X=1)$.

Soln. $P(X=1) = P\left(\{\text{Choosing exactly one black pen}\}\right)$

$$= \frac{\binom{5}{1}\binom{3}{2}}{\binom{8}{3}} = \frac{5 \cdot 3}{\frac{8 \cdot 7 \cdot \cancel{6}}{\cancel{3!}}} = \frac{15}{56}.$$

Exc. In the above example, let Y be the number of red pens within the taken ones.

(1) What are the possible values of Y.

(2) Find $P(Y=1)$.

Def. A distribution of a r.v. X is a table or a formula or a graph that gives or shows all possible values of X together with a p.d.f. of X.

Ex. Consider the following r.v. with the given p.d.f.. Find

(1) K.

(2) $P(3 \le x < 5)$

(3) $P(X \text{ is even} \mid x > 1)$

(4) $P(x \text{ is odd or } x > 3)$

x	$P(x)$
1	K
2	0.05
3	0.25
4	0.1
5	0.35
6	0.1

Soln. (1) $k = 1 - (0.05 + 0.25 + 0.1 + 0.35 + 0.1) = 0.15.$

(2) $P(3 \leq X \lneq 5) = P(X=3) + P(X=4)$

$$= 0.25 + 0.1$$

$$= 0.35.$$

(3) $$P(x \text{ is even} \mid x>1) = \frac{P(x \text{ is even and } x>1)}{P(x>1)}$$

$$= \frac{P(x=2) + P(x=4) + P(x=6)}{P(x>1)}$$

$$= \frac{0.05 + 0.1 + 0.1}{0.05 + 0.25 + 0.1 + 0.35 + 0.1}$$

$$= \cdots .$$

(4) $$P(x \text{ is odd or } x>3) = P(x \text{ is odd}) + P(x>3) - P(x \text{ is odd and } x>3)$$

$$= 0.75 + 0.55 - 0.35$$

$$= 0.95.$$

Fact: Assume that X is a continuous r.v..
Then p.d.f. $P(x)$ satisfies the following conditions :-

(1) $P(x) = \begin{cases} 0 & \text{if } x \notin R_X, \\ +\text{ve} & \text{if } x \in R_X. \end{cases}$ So, $P(x) \geq 0, \forall x$.

(2) $\int_{R_X} P(x)\, dx = 1$.

<u>Def.</u> Let X be a r.v. with p.d.f. $P(x)$ and range R_X.

(1) The <u>expectation</u> or <u>expected value</u> or <u>mean</u> of X, is denoted by $E(X)$ and is defined as

$$E(X) = \begin{cases} \sum_{x \in R_X} x P(x) = \sum_{i=1}^{\infty} x_i P(x_i) & \text{if } X \text{ is a discrete r.v.} \\ \int_{R_X} x P(x)\, dx & \text{if } X \text{ is a continuous r.v.} \end{cases}$$

(provided that the sum/integral exists).

(2) The <u>variance</u> of X is denoted by $\text{var}(X)$ or σ_X^2 and is defined as

$$\text{var}(X) = E(X - E(X))^2 = E(X^2) - (E(X))^2.$$

(3) Standard deviation $\sigma_X = \sqrt{var(X)}$.

Fact: $E(X^n) = \sum x^n P(x)$ (or $\int x^n P(x)\,dx$ for cts)

In particular, $E(X^2) = \sum x^2 P(X)$ (or $\int x^2 P(x)\,dx$ for cts)

Ex. Consider the following r.v. and its p.d.f.

x	$P(x)$	$xP(x)$	x^2	$x^2 P(x)$
1	0.1	0.1	1	0.1
2	0.2	0.4	4	0.8
3	0.3	0.9	9	2.7
4	0.4	1.6	16	8.4

Find (1) Var (X) (2) Standard deviation σ_X

Soln. $E(X) = \sum x P(x) = 0.1 + 0.4 + 0.9 + 1.6 = 3$

$E(X^2) = \sum x^2 P(x) = 0.1 + 0.8 + 2.7 + 6.4 = 10$.

(1) $Var(X) = E(X^2) - (E(X))^2 = 10 - (3)^2 = 1$.

(2) $\sigma_X = \sqrt{var(X)} = \sqrt{1} = 1$.

Ex. Assume that the expectation of the following is 2.9, find K and L.

x	$P(x)$
1	0.1
2	0.3
3	K
4	L

Soln. $K + L = 1 - (0.1 + 0.3) = 0.6$ —— $(*)$

$E(X) = 0.1 + 0.6 + 3K + 4L$

Then $3K + L = 2.9 - 0.1 - 0.6 = 2.2$.

$\boxed{(*)} \Rightarrow 3(0.6 - L) + 4L = 2.2$

$\Rightarrow L = 2.2 - 1.8 = 0.4$

$\Rightarrow K = 0.6 - 0.4 = 0.2$. Done.

Searching keywords:

- Univariate random variable.
- Distribution of a r.v.
- Expectation, expected value.
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx

For any comments or concerns, please use my email to contact me.

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