Integration: Review of formulas.

1) $\int k\,dx = kx + C$, k is a constant.

2) $\int x^r\,dx = \frac{1}{r+1}x^{r+1} + C$, $r \neq -1$.

3) $\int \frac{dx}{x} = \ln|x| + C$.

4) $\int e^x\,dx = e^x + C$.

5) $\int p^x\,dx = \frac{p^x}{\ln p} + C$, $p \neq$ is a positive constant.

6) $\int \sin x\,dx = -\cos x + C$.

7) $\int \cos x\,dx = \sin x + C$.

8) $\int \tan x\,dx = \ln|\sec x| + C = -\ln|\cos x| + C$.

9) $\int \cot x\,dx = \ln|\sin x| + C$.

10) $\int \sec x\,dx = \ln|\sec x + \tan x| + C$.

11) $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$.

12) $\int \sec x \tan x\,dx = \sec x + C$.

13) $\int \csc x \cot x\,dx = -\csc x + C$.

14) $\int \sec^2 x\,dx = \tan x + C$.

15) $\int \csc^2 x\, dx = -\cot x + C.$

$$\left.\begin{aligned}
&16)\ \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C \\
&17)\ \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \\
&18)\ \int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{|x|}{a}\right) + C
\end{aligned}\right\} \text{$a>0$ is a constant.}$$

19) $\int \sinh x\, dx = \cosh x + C$

20) $\int \cosh x\, dx = \sinh x + C.$

<u>Example</u> $\int (x^2-5)^2\, dx = \int (x^4 - 10x^2 + 25)\, dx$

$$= \frac{1}{5}x^5 - \frac{10}{3}x^3 + 25x + C.$$

<u>Integration by substitution: A brief review.</u>

How to find $I = \int f(g(x))\, g'(x)\, dx$ ☹

Set $u = g(x)$, then $du = g'(x)\, dx.$

Then $I = \int f(u)\, du.$ ☺

Example: Evaluate

(1) $\int x \cot x^2 \, dx$.

Soln. Set $u = x^2$, then $du = 2x\,dx$.

$$= \int \cancel{x} \cot u \frac{du}{2\cancel{x}} = \frac{1}{2}\int \cot u \, du = \frac{1}{2}\ln|\sin u| + C = \frac{1}{2}\ln|\sin x^2| + C$$

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(2) $\int \frac{e^x}{(e^x+3)^{1/3}} \, dx$.

Soln. Let $u = e^x + 3$, then $du = e^x \, dx$

$$= \int \frac{e^x}{u^{1/3}} \frac{du}{e^x} = \frac{u^{2/3}}{2/3} + C = \frac{3}{2}\left(e^x+3\right)^{2/3} + C.$$

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(3) $I = \int \frac{dx}{\sqrt{-5+6x-x^2}}$.

Soln. Note that

$$\begin{aligned} -5+6x-x^2 &= -5+6x-x^2-9+9 \\ &= -5-(x^2-6x+9)+9 \\ &= 4-(x^2-6x+9) \\ &= 4-(x-3)^2. \end{aligned}$$

Then $I = \int \frac{dx}{\sqrt{4-(x-3)^2}} = \int \frac{dx}{2\sqrt{1-\left(\frac{x-3}{2}\right)^2}}$

Let $u = \frac{x-3}{2}$, then $du = \frac{1}{2}dx$

$= \int \frac{\cancel{2}\, du}{\cancel{2}\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1}\left(\frac{x-3}{2}\right) + C.$

Exc Show that $\underset{I}{\int} \frac{4x+1}{2x^2+4x+10}\,dx = \ln\left[\left(\frac{x+1}{2}\right)^2+1\right] - \frac{3}{4}\tan^{-1}\left(\frac{x+1}{2}\right) + C.$

Proof. $I = \int \frac{4x+1}{2(x+1)^2+8}\,dx = \frac{1}{8}\int \frac{4x+1}{\left(\frac{x+1}{2}\right)^2+1}\,dx$

Let $u = \frac{x+1}{2}$, then $du = \frac{1}{2}dx$ and $x = 2u-1$.

$= \frac{1}{4}\int \frac{4(2u-1)+1}{u^2+1}\,du = \frac{1}{4}\int \frac{8u-3}{u^2+1}\,du$

$= \frac{\cancel{4}}{\cancel{4}}\int \frac{2u}{u^2+1}\,du - \frac{3}{4}\int \frac{du}{u^2+1}$

$= \ln(u^2+1) - \frac{3}{4}\tan^{-1} u + C.$

$= \ln\left(\left(\frac{x+1}{2}\right)^2+1\right) - \frac{3}{4}\tan^{-1}\left(\frac{x+1}{2}\right) + C.$

This lecture: Integration- Introduction with some integration formulas.

Next lecture: Integration by parts.

Searching keywords:

- Evaluate the integral احسب التكامل
- Integration by substitution التكامل بالتعويض
- The University of Jordan الجامعة الأردنية
- Calculus II 2 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/102.aspx

For any comments or concerns, please use my email to contact me.

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