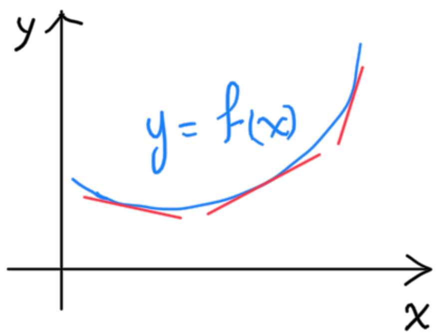
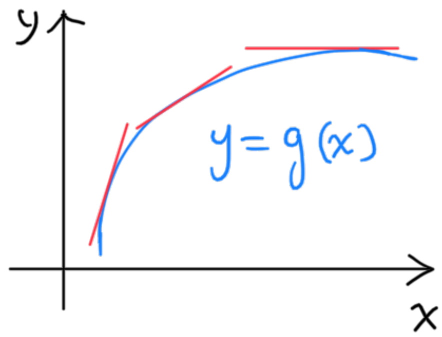
Concavity

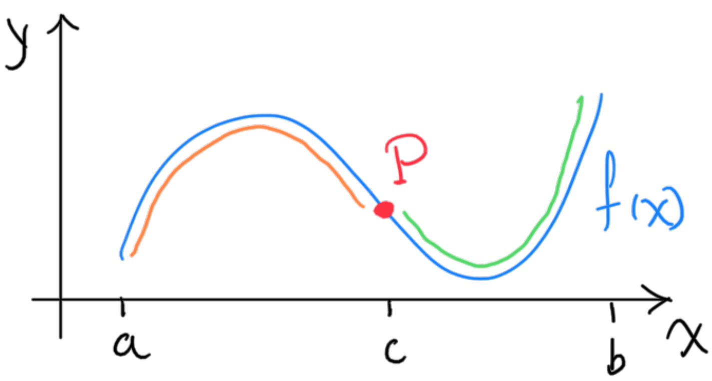




f(x) is an increasing func.
The slope of f(x) increases
f(x) is concave up

g(x) is a decreasing func.
The slope of g(x) decreases
g(x) is concave down.

<u>Def.</u> If the graph of f lies above all of its tangents on an interval I, then it is called <u>concave upward</u> on I. If the graph of f lies below all of its tangents on I, it is called <u>concave downward</u> on I.



Here the graph of f(x) is <u>concave down</u> on (a, c) and <u>concave up</u> on (c, b).

Def. A point P on a curve $y = f(x)$ is called an inflection point if f is cts there and the curve changes from concave up to concave down or from concave down to concave up at P.

Concavity test.

(A) If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

(B) If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.

Ex. Determine where the graph of $f(x)$ is concave up and concave down, and find any inflection points.

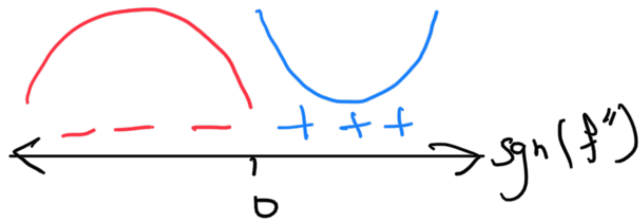
(1) $f(x) = x^5 - 5x$.

Soln. $f'(x) = 5x^4 - 5$, $f''(x) = 20x^3$.

$f''(x) = 0$ when $x = 0$.

$f''(x) > 0$ on $(0, \infty)$

$f''(x) < 0$ on $(-\infty, 0)$.



Thus, f is concave up on $(0, \infty)$,

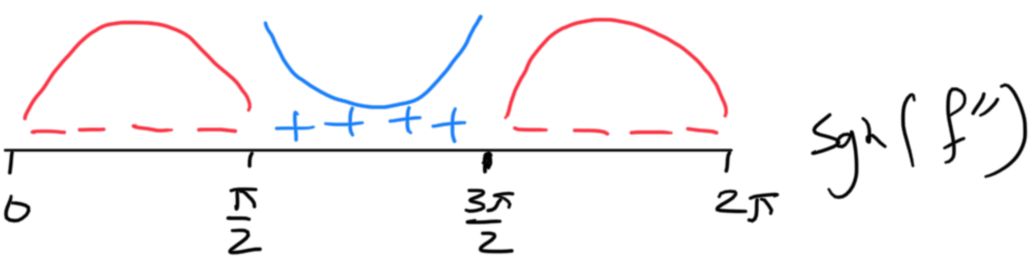
and concave down on $(-\infty, 0)$.

The point $(0, f(0)) = (0, 0)$ is the point of inflection.

(2) $f(x) = x + \cos x, \quad x \in [0, 2\pi]$.

<u>Soln.</u> $f'(x) = 1 - \sin x, \quad f''(x) = -\cos x$.

$f''(x) = 0$ when $x = \pi/2, 3\pi/2$.



Thus, $f(x)$ is concave up on $(\pi/2, 3\pi/2)$,

and concave down on $(0, \pi/2) \cup (3\pi/2, 2\pi)$.

The points $(\pi/2, f(\pi/2)) = (\pi/2, \pi/2)$

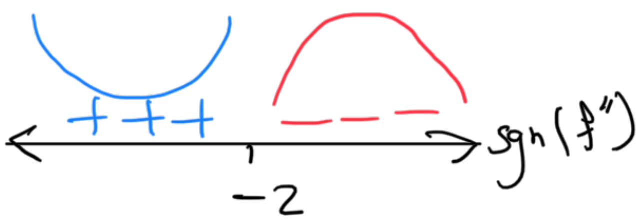
and $(3\pi/2, f(3\pi/2)) = (3\pi/2, 3\pi/2)$

are the points of inflection.

(3) $f(x) = (x+2)^{1/5} + 4$.

<u>Soln.</u> $f'(x) = \frac{1}{5}(x+2)^{-4/5}$, $f''(x) = \frac{-4}{25}(x+2)^{-9/5}$.

$f''(-2)$ is undefined.



f is concave up on $(-\infty, -2)$.

f is concave down on $(-2, \infty)$.

The point $(-2, f(-2))$ is the inflection point.

(4) $f(x) = e^{1/x}$. Exc.

<u>FINAL ANS.</u> f is concave down on $(-\infty, -1/2)$.

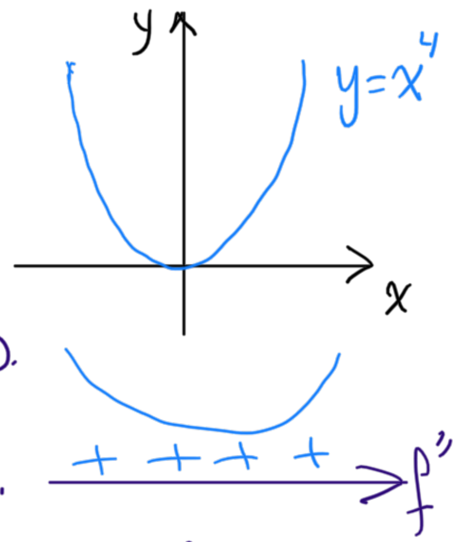
f is concave up on $(-1/2, 0) \cup (0, \infty)$.

The inflection point is $(-1/2, e^{-2})$.

<u>Thm.</u> If the point $(c, f(c))$ is a point of inflection, then $f''(c) = 0$ or $f''(c)$ DNE.

Remark: The converse of the above theorem is not true in general.

Ex. Take $f(x) = x^4$



$f'(x) = 4x^3$,

$f''(x) = 12x^2 > 0$ for all $x \neq 0$.

$\therefore f$ is concave up for $x \neq 0$.

Note that $f''(0) = 0$, but f has no inflec. pt.

Searching keywords:

- Concavity, concave up, concave down التقعر، مقعر للأعلى، مقعر للأدنى
- Inflection point نقطة انعطاف
- The University of Jordan الجامعة الأردنية
- Calculus I 1 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/101.aspx

For any comments or concerns, please use my email to contact me.

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