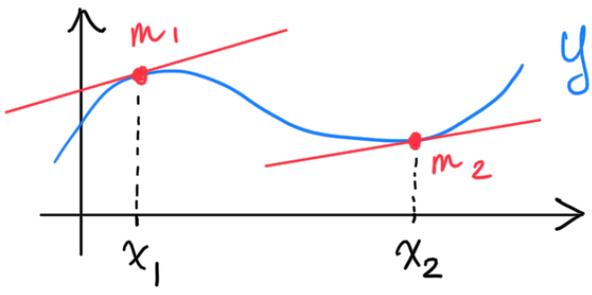
The derivative as a rate of change

Given a diff. func. $y = f(x)$, the rate of change of y with respect to (w.r.t.) x is the slope $\boxed{\frac{dy}{dx} = f'(x)}$.

Note: This rate of change can vary from a point to another.



$y = f(x)$

$m_1 = f'(x_1)$

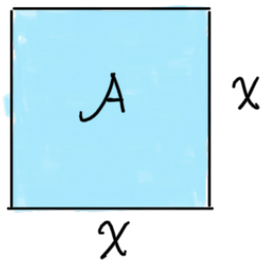
$m_2 = f'(x_2)$.

Ex. Let A be the area of a square of side x. Find the rate of change of A w.r.t. x.

Soln. Area $A(x) = x^2$.

Then the rate of change of A w.r.t x is

$$\frac{d}{dx} A = (x^2)' = 2x.$$

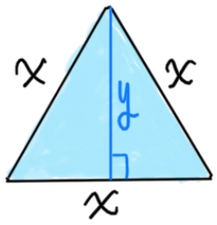


For instance

when $x = \frac{1}{4}$, the rate of change is $1/2$.

when $x = 1$, " " " " " 2. That is, the area is changing at twice the rate of x.

Ex. Let A be the area of an equilateral triangle of side x. Find the value(s) of x at which the rate of change of A is $\sqrt{3}$.



Soln. $x^2 = y^2 + \left(\frac{x}{2}\right)^2$.

Then $y^2 = x^2 - \frac{x^2}{4} = \frac{3}{4}x^2$.

It follows that $y = \frac{\sqrt{3}}{2}x$.

Area $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$.

The rate of change of A w.r.t. x is $\frac{dA}{dx} = \frac{\sqrt{3}}{2}x$.

Now, $\frac{dA}{dx} = \sqrt{3}$ when $\frac{\sqrt{3}}{2}x = \sqrt{3}$. So, $\boxed{x = 2.}$

Velocity, acceleration and speed.

Def. Let $x(t)$ be the position of an object at time t. We have

(1) If $x(t)$ is diff., then the velocity at time t is $\boxed{v(t) = x'(t)}$.

(2) If $v(t)$ is diff., then the acceleration at time t is $\boxed{a(t) = v'(t) = x''(t)}$.

(3) The speed of the object at time t is $|v(t)|$.

Ex. Suppose that the height of a skydiver (she/he) at time t seconds after jumping from an airplane is given by $f(t) = 640 - 20t - 16t^2$ feet.

Find: (1) The person's velocity at time t.

(2) ; = acceleration ; = ;.

Soln. (1) $v(t) = f'(t) = -20 - 32t$ ft/sec.

(2) $a(t) = v'(t) = -32$ ft/sec^2.

Exc. The equation of motion of a particle is

$$s = s(t) = 2t^3 - 5t^2 + 3t + 4 \quad \text{cm}$$

What is the acceleration after 2 seconds?

FINAL ANS; $a(2) = 14 \ \text{cm/s}^2$.

Searching keywords:

- Rate of change معدل التغير
- Velocity, acceleration السرعة، التسارع
- The University of Jordan الجامعة الأردنية
- Calculus I 1 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/101.aspx

For any comments or concerns, please use my email to contact me.

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