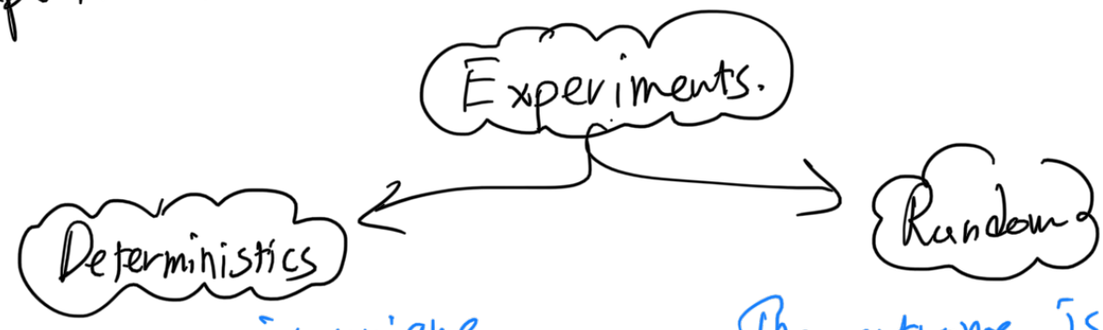
Chapter 2: Elements of Probability.

Random Experiment.

Def. A performance of an experiment is called a **trial** of the experiment.

Def. An observed result on a trial of the experiment is called an **outcome**.



The outcome is unique and certain.
تكرار التجربة لا يغير النتيجة

The outcome is not unique and several outcomes are possible.
عند تكرار التجربة، النتيجة قد تتغير عشوائيًا

Examples:

(1) The result of boiling temperature of pure water at sea level. ← Deterministic exper.

(2) The result of tossing a coin ← Random exper.
(Not sure whether a head or tail will arise).

Def. The **sample space** is the collection of all possible outcomes that might be observed from a random experiment. This set is denoted by Ω.

Def. An **event** is a subset of a sample space.

Ex. If we roll a die one time, then
Random experiment is rolling a die.
Sample space is $\Omega = \{1,2,3,4,5,6\}$.
The event of obtaining odd numbers is corresponding to the subset $A = \{1,3,5\}$.

Ex. If we roll a die and then toss a coin, then
Random experiment is rolling a die and tossing a coin.
Sample space is $\Omega = \{(1,H), \dots, (6,H), (1,T), \dots, (6,T)\}$.

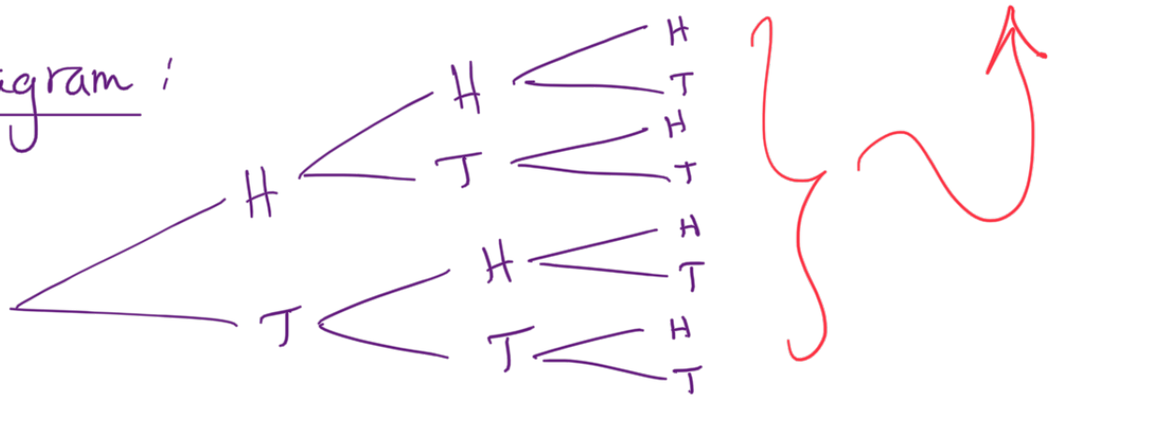
Def. An event occurs or happens if at least one of its elements occurs.

Def. An event is called simple or elementary if it contains exactly one outcome of the experiment.

Def. The impossible event, denoted by ϕ, is the event that contains no outcomes and therefore cannot occur.

Ex. If we toss three coins, then
Sample space is $\Omega = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$

Trea diagram:



The event of obtaining at least two heads is

$A = \{(H,H,H), (H,H,T), (H,T,H), (T,H,H)\}$.

The event of obtaining at least three heads is

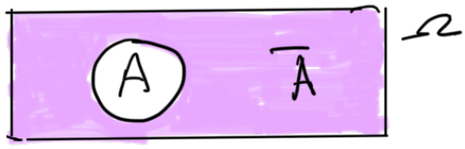
$B = \{(H,H,H)\}$.

Note that A is not simple, while B is a simple event.

Theoretical notations of events

Let Ω be the sample space of some random experiment and let A and B be two events in Ω. Then:

(1) $\bar{A}$: The complement of A.



All outcomes in Ω which are not in A.

(2) $A \subset B$; A is a proper subset of B

Every outcome in A belongs to B.

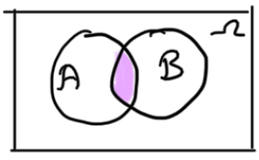
(3) $A = B$: A and B are equal.

Outcomes in A and those in B are identical.

(4) $A \subseteq B$: A is a subset of B.

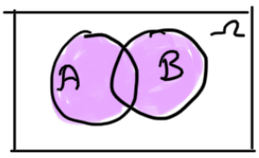
$A \subset B$ or $A = B$.

(5) $A \cap B$: The intersection of A and B



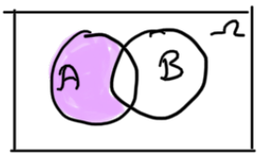
The outcomes that belong to both A and B.

(6) $A \cup B$: The union of A and B



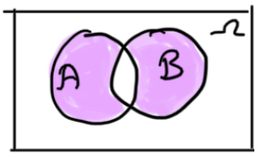
The outcomes that belong to A or to B.

(7) $A - B = A \cap \overline{B}$



The outcomes that are in A but not in B.

(8) $A \Delta B$: The symmetric difference of A and B.



$A \Delta B = (A - B) \cup (B - A)$

(9) $A \times B$: The Cartesian product of A and B

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

Ex. If $A = \{1, 2\}$ and $B = \{3, 4\}$, then

$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$.

Def. Two events A and B are called mutually exclusive or disjoint if $A \cap B = \phi$.

Fact: Let A, B and C be three events, then

(1) $\overline{\overline{A}} = A$.

(2) $A \cup A = A \cap A = A$.

(3) $A \cap \phi = \phi$, $A \cap \overline{A} = \phi$

(4) $A \cap \Omega = A$, $A \cup \phi = A$

(5) $A \cup \Omega = \Omega$, $A \cup \overline{A} = \Omega$.

(6) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

(7) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ and $\overline{A \cap B} = \overline{A} \cup \overline{B}$. } De-Morgan's laws

In general, $\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$ and $\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$.

Searching keywords:

- Random experiment, deterministic.
- Trial, outcome, sample space.
- Events, union, intersection, complement.
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx

For any comments or concerns, please use my email to contact me.

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