

Measures of Variation

For raw data:

(1) Range.

The sample range is the difference between maximum and minimum observations.

(2) Interquartile range.

The sample interquartile range is the difference between the third and first quartiles.

$$IQR = Q_3 - Q_1.$$

(3) Standard deviation. *الانحراف المعياري*

Let x_1, x_2, \dots, x_n be a sample of size n . Then the sample standard deviation (Std.) is given by

$$s' = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad \text{or} \quad s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

We will use this formula.

Remark: It can be shown that

$$s' = \sqrt{\frac{1}{n-1} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)} \quad \text{or} \quad s = \sqrt{\frac{1}{n} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)}$$

(4) Variance σ^2 ,

Variance is S^2 . So $\text{Std.} = \sqrt{\text{Variance}}$.

(5) Mean deviation.

The mean deviation (M.D.) is given by

$$\text{M.D.} = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}|.$$

(6) Median deviation.

The median deviation is given by

$$\text{Median Deviation} = \frac{1}{n} \sum_{i=1}^n |x_i - \text{Median}|.$$

Ex. Given the following data

1, 14, 15, 9, 4, 3.

Find the range, mean, variance, standard deviation, mean deviation, median, median deviation.

Soln. Sample size is $n = 6$.

Range = Max Obs. - Min Obs. = $15 - 1 = 14$.

$$\text{Mean } \bar{x} = \frac{1}{6} \sum_{i=1}^6 x_i = \frac{1}{6} (1 + 14 + 15 + 9 + 4 + 3) = \frac{46}{6} = 7.667.$$

$$\text{Variance } S^2 = \frac{1}{n-1} \sum_{i=1}^6 (x_i - \bar{x})^2$$

$$= \frac{1}{5} [(1 - 7.667)^2 + (14 - 7.667)^2 + \dots + (3 - 7.667)^2]$$

$$= 35.667.$$

Standard deviation is $S = \sqrt{S^2} = \sqrt{35.067} = 5.92$.

Mean deviation is

$$M.D. = \frac{1}{n} \sum_{i=1}^6 |x_i - \bar{x}| = \frac{1}{6} [(1-7.667) + (14-7.667) + \dots + (3-7.667)]$$

$$\text{Median} = \frac{1}{2} (X_3 + X_4) = \frac{13}{2} = 6.5$$

as n is even 1, 3, 4, 9, 14, 15

Median deviation is

$$\frac{1}{n} \sum_{i=1}^n |x_i - \text{Median}| = \frac{1}{6} [(1-6.5) + (14-6.5) + \dots + (3-6.5)]$$

= \dots

Ex. Given the following data.

9, 9, 9, 9, 9, 9, 9, 9, 9, 9.

Find the sample variance.

Soln. $S^2 = \frac{1}{9} [(9-9)^2 + (9-9)^2 + \dots + (9-9)^2] = 0.$

For grouped data:

(1) Standard deviation

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 f_i} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n x_i^2 f_i - n\bar{x}^2}$$

where x_i is the class center and f_i is its frequency.

(2) Mean deviation

$$M.D. = \frac{1}{n} \sum_{i=1}^n |x_i - \bar{x}| f_i$$

(3) Median deviation

$$\text{Median deviation} = \frac{1}{n} \sum_{i=1}^n |x_i - \text{Median}| f_i$$

Ex- The following table provides the calculation of Std., mean deviation, and median deviation.

| Actual Classes | Classes | Freq. (f_i) | Midpoint (x_i) | $x_i - \bar{X}$ | $(x_i - \bar{X})^2$ | $(x_i - \bar{X})^2 f_i$ | $ x_i - \bar{X} $ | $ x_i - \bar{X} f_i$ | $ x_i - \text{Median} $ | $ x_i - \text{Median} f_i$ |
|----------------|---------|-----------------|--------------------|-----------------|---------------------|-------------------------|-------------------|-----------------------|-------------------------|-----------------------------|
| 9.5 - 19.5 | 10 - 19 | 4 | 14.5 | -9.71 | 94.283 | 388.3 | 9.71 | 78.8223 | 20 | 80 |
| 19.5 - 29.5 | 20 - 29 | 7 | 24.5 | -9.71 | 94.20 | 659.429 | 9.71 | 67.9412 | 10 | 70 |
| 29.5 - 39.5 | 30 - 39 | 12 | 34.5 | 0.294 | 0.087 | 1.03806 | 0.294 | 3.52941 | 0 | 0 |
| 39.5 - 49.5 | 40 - 49 | 8 | 44.5 | 10.29 | 105.97 | 847.751 | 10.29 | 82.353 | 10 | 80 |
| 49.5 - 59.5 | 50 - 59 | 3 | 54.5 | 20.29 | 411.85 | 1235.55 | 20.29 | 60.8824 | 20 | 60 |

$$n = 34$$

$$\text{Mean } \bar{X} = 34.2059$$

$$\text{Std.} = 11.41$$

$$\text{Mean deviation} = 8.63322$$

$$\text{Median} = 34.5$$

$$\text{Median deviation} = 8.53$$

Searching keywords:

- Measure of variation.
- Standard deviation, variance.
- Mean deviation, median deviation.
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the course website

<http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx>

For any comments or concerns, please use my email to contact me.



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