Hypothesis testing.

Last time, we found that the first procedure of making statistical inference about a population parameter is the confidence interval. The second procedure is the hypothesis testing, which will be studied today!

In a test of hypothesis, we test a certain given claim (or belief) about a population parameter. We use some information obtained from samples to check whether or not a given claim about a population parameter is true.

Ex. In a criminal trial, when a person is accused of a community crime, he or she faces a trial. Based on the available evidence, the judge will make one of two possible decisions:-

1) The person is not guilty.

2) The person is guilty.

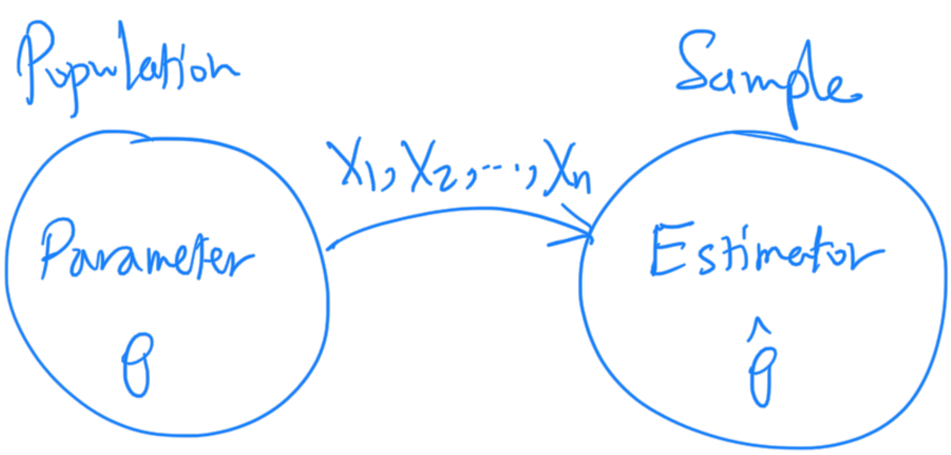
The judge must take his decision based on the information presented by both the prosecution and defence.

Null hypothesis

The null hypothesis is a statement about a population parameter that is assumed to be true until it is declared false.

Alternative hypothesis

The alternative hypothesis is a statement about a population parameter that will be true if the null hypothesis is false.



$H_0 : \theta = \hat{\theta}$ vs. $H_1 : \theta > \hat{\theta}$, or $\theta < \hat{\theta}$, or $\theta \neq \hat{\theta}$.

In the above example, the judge conducts a hypothesis test as follows:

H_0: The person is not guilty.

H_1: The person is guilty.

There are two possible decisions:

Reject H_0 or Do not reject H_0.

The judge's decision is not always correct, which could lead to 2 errors. The following table shows this.

Decision	H_0 True	H_0 False
Reject H_0	Type I error	✓
Do not reject H_0	✓	Type II error

Type I error: Made by punishing an innocent person.

Type II error: Made by setting a guilty person free.

Type I error

A type I error occurs when rejecting a true H_0.

Type II error

A type II error occurs when a false H_0 is not rejected.

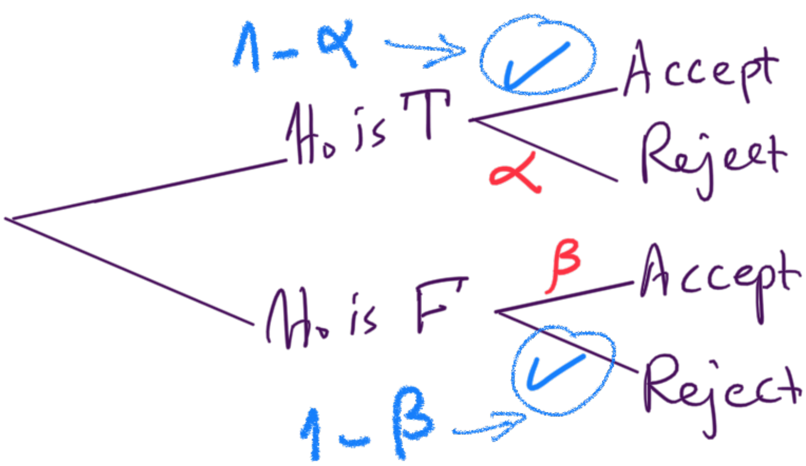
We define:

$\alpha = P(\text{type I error}) = P(\text{Reject } H_0 \mid H_0 \text{ is true})$.

$\beta = P(\text{type II error}) = P(\text{Accept } H_0 \mid H_0 \text{ is false})$.

Here, α is called the <u>significance level</u>,

and $1-\beta$ 〃 〃 〃 <u>power of the test</u>.



Def. A <u>test statistic</u> is a random variable that is calculated from sample data and used in a hypothesis test.

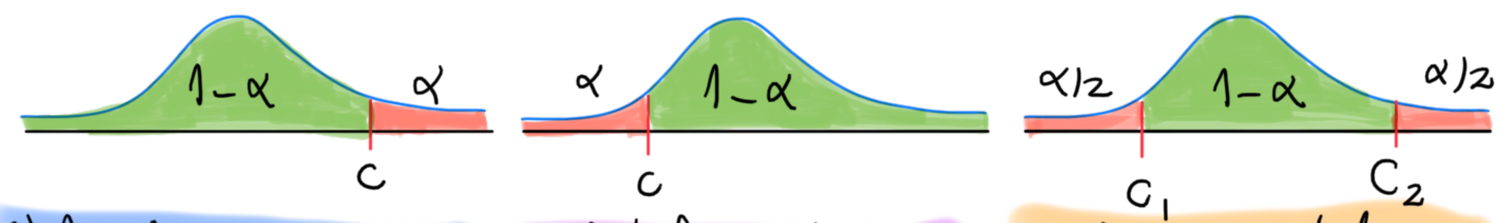
You can use test statistics to determine whether to reject H_0. The test statistic compares our data with what is expected under H_0.

1) $\theta > \hat{\theta}$
2) $\theta < \hat{\theta}$
3) $\theta \neq \hat{\theta}$

Non-rejection region.

Rejection region.

c Critical value.



1) A right-tailed test

2) A left-tailed test

3) A two-tailed test

A one tailed test.

<u>Statiscal test for μ.</u>

$$H_0 : \mu = \mu_0 \quad \text{v.s.} \quad H_1 : \begin{array}{l} \mu > \mu_0 \\ \mu < \mu_0 \\ \mu \neq \mu_0 \end{array}$$

The test statistic is:

i) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} \sim N(0,1)$ if σ is known.

ii) $T = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim T(n-1)$ if σ is unknown

iii) $Z = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim N(0,1)$ if σ is unknown but $n \geq 30$.

Ex. The mean cholesterol levels in a general population are normally distributed. A sample of 16 persons is taken under a test with sample mean $\bar{X} = 220$ mg/dl and standard deviation $S = 25$ mg/dl. Test at 1% significance level that the mean cholesterol level is less than 230 mg/dl.

Soln. $n = 16$, $\bar{X} = 220$, $S = 25$, $\alpha = 0.01$.

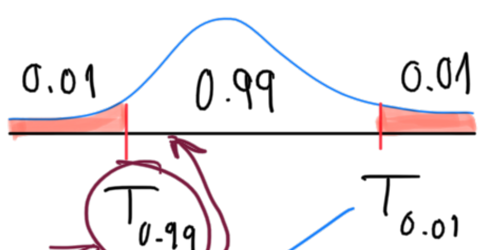
$H_0: \mu = 230$ v.s. $H_1: \mu < 230$.

Test statistic is

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = \frac{220 - 230}{25/\sqrt{16}} = -1.6.$$

d.f. = 15

Because $T = -1.6 > T_\alpha = -2.602$, we do not reject H_0.



$\Rightarrow = 2.602$ from tables.

So $T_{0.99} = -2.602$.

-2.602

-1.6

Ex. A random sample of 400 people with a professional degree taken showed that their mean monthly salary is 450 JD with a standard deviation of 100 JD. Test at 5% significance level that the mean monthly salary is different from 460 JD.

Soln. $n=400$, $\bar{X}=450$, $S=100$, $\alpha=0.05$, $\alpha/2=0.025$.

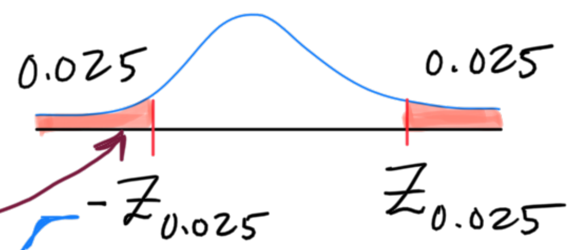
$H_0: \mu=460$ v.s. $H_1: \mu\neq 460$

Test statistic is

$$Z=\frac{\bar{X}-\mu_0}{S/\sqrt{n}}=\frac{450-460}{100/\sqrt{400}}=-2.$$

Because $Z=-2<-Z_{0.05}=-1.96$ we reject H_0.

The value -2



$=-1.96$ (from tables).

So $Z_{0.025}=1.96$.

Statistical test for σ^2

$H_0: \sigma^2=\sigma_0^2$ v.s. H_1: $\sigma^2>\sigma_0^2$, $\sigma^2<\sigma_0^2$, or $\sigma^2\neq\sigma_0^2$.

The test statistic is:

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} \sim \chi^2(n-1).$$

Ex. Quality-control engineer wishes to study the weight variation of a new product. A sample of 10 items is taken and provided

$$\bar{X} = 0.6 \text{ kgs} \quad \text{and} \quad S = 0.4 \text{ kgs}.$$

Assume that the distribution of the weights can be modeled as a normal distribution.

a) Test $H_0: \sigma^2 = 0.5$ v.s. $H_1: \sigma^2 > 0.5$.
   Use $\alpha = 0.025$.

b) Test $H_0: \sigma = 0.74$ v.s. $\sigma \neq 0.74$.
   Use $\alpha = 0.10$.

Soln. $n = 10$, $\bar{X} = 0.6$, $S = 0.4$.

a) $H_0: \sigma = 0.5$ v.s. $H_1: \sigma > 0.5$.
   $\alpha = 0.025$.

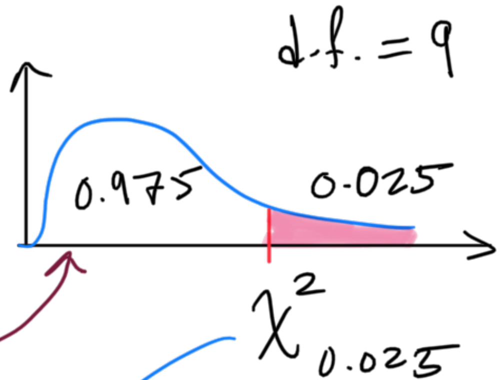
Test statistic is

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(10-1)(0.4)^2}{0.5} = 2.88$$

Because $\chi^2 = 2.88 < \chi^2_{0.025}$

$= 2.88$

we don't reject H_0.



$= 19.0228$ (from tables).

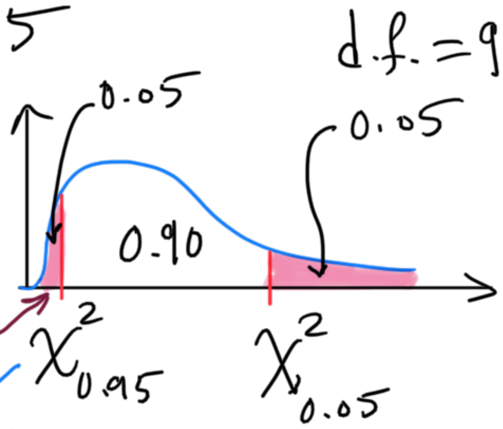
b) $\sigma = 0.74$, so $\sigma^2 = 0.55$.

or $H_0: \sigma^2 = 0.55$ v.s. $\sigma^2 \neq 0.55$

Test statistic is

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{(10-1)(0.4)^2}{0.55} = 2.62$$

$\alpha = 0.10$, so $\alpha/2 = 0.05$



Because

$\chi^2 = 2.62 < \chi^2_{0.95} = 3.3251$

$= 3.3251$ (from tables)

we reject H_0.

<u>Statiscal test for p.</u>

$H_0: p = p_0$ v.s. H_1: $p > p_0$,

$p < p_0$,

or $p \neq p_0$

The test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1).$$

Ex. It was believed in the Arab World that 50% of persons are smoking. During the year 2000, a sample of 1000 persons showed that the number of smokers is 620. Can you conclude that the proportion of smokers is different from 50%? Use $\alpha = 0.01$.

Soln. $n = 1000$, $X = 620$, $\hat{p} = \frac{X}{n} = \frac{620}{1000} = 0.62$.

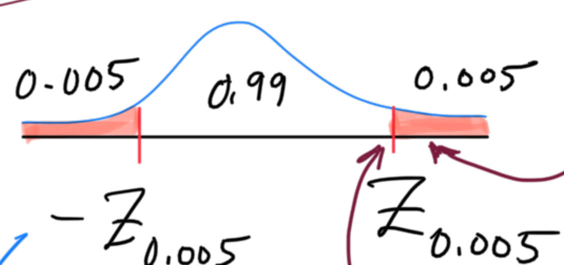
$H_0 : p = 0.50$ v.s. $H_1 : p \neq 0.50$

Test statistic is

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.62 - 0.50}{\sqrt{\frac{0.5(1-0.50)}{100}}} = 7.59$$

$\alpha = 0.01$, so $\frac{\alpha}{2} = 0.05$.

Because $Z = 7.59 > Z_{0.05} = 2.57$, we reject H_0.



$= -2.57$ (from tables).

So $Z_{0.005} = 2.57$.

Relationship between tests and confidence intervals.

<u>Fact</u>: Let (L, U) be a $(1-\alpha)100\%$ C.I. for unknown parameter θ. The null hypothesis $H_0: \theta = \theta_0$ is rejected against $H_1: \theta \neq \theta_0$ at significance level α if θ_0 does not belong to (L, U).

<u>Ex.</u> A random sample of 8 observations was taken from a normal population. The sample mean and standard deviation are $\bar{X} = 70$ and $S = 20$. Find a 95% C.I. for μ and test at 5% significance level

$$H_0: \mu = 80 \quad \text{v.s.} \quad H_1: \mu \neq 80.$$

<u>Soln.</u> $n = 8$, $\bar{X} = 70$, $S = 20$.

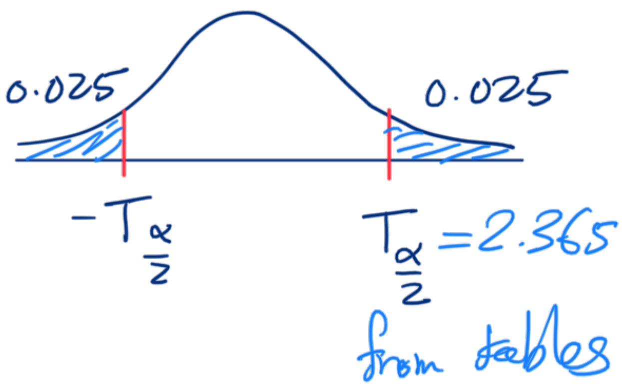
$1 - \alpha = 0.95$, so $\alpha = 0.05$ and hence $\frac{\alpha}{2} = 0.025$.

The 95% C.I. for μ is

$$\bar{X} \pm T_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}} = 70 \pm 2.365\left(\frac{20}{\sqrt{8}}\right),$$

or equivalently, $(53.29, 86.71)$.

As $80 \in (53.29, 86.71)$, we don't reject H_0.



Searching keywords:

- Estimation of confidence interval
- Hypothesis testing, test statistic, test statistics
- Null hypothesis
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the textbook in the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx

For any comments or concerns, please use my email to contact me.

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