The distribution of the sample proportion.

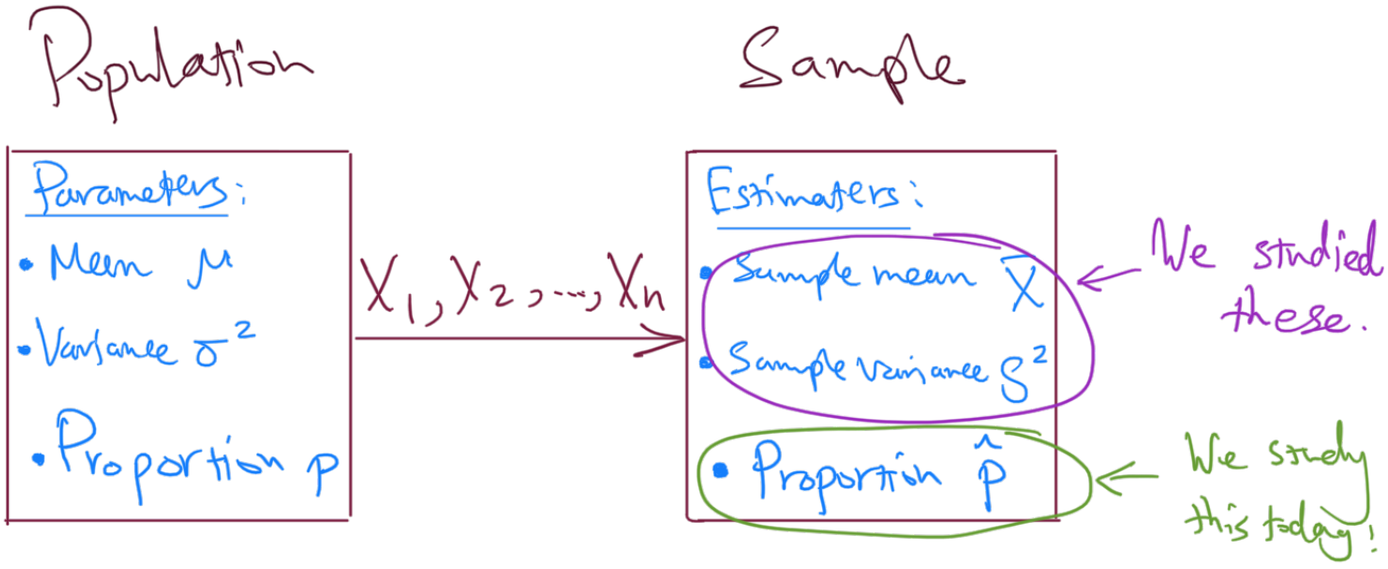
There are many practical problems in which we are interested in getting information about proportion p of individuals or items in a certain population who have a specified property or characteristics.

For example, we might be interested in the proportion of

- smokers among the university students.
- defectives in the product of a certain factory.

We estimate a certain proportion p by taking a sample of size n and recording the number of items X with the characteristics under consideration.

Then we use $\hat{p} = \frac{X}{n}$ as an estimate of p.



If the sample size n is large, we can show that the distribution of $\hat{p}$ can be approximated by the normal distribution with mean p and variance $p(1-p)/n$.

That is, $\hat{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$

or $Z = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$.

Ex. Suppose that 10% of a certain production are defectives. If 400 items are drawn from the production, what is the probability that the sample population will be

a) more than 12%?

b) between 9% and 11%?

Soln. $p = 0.10$, $1 - p = 0.90$, $n = 400$.

$\hat{p} \sim N(p, p(1-p)/n)$

$\hat{p} \sim N(0.10, (0.10)(0.90)/400)$

$\hat{p} \sim N(0.10, (3/200)^2)$.

a) $P(\hat{p} > 0.12) = P\left(Z > \frac{0.12 - 0.10}{3/200}\right)$

$= P(Z > 1.23)$

$= 1 - P(Z \leq 1.23)$

$= 1 - 0.9082$

$= 0.0918.$

b) $P(0.09 < \hat{p} < 0.11) = P\left(\frac{0.09 - 0.10}{3/200} < Z < \frac{0.11 - 0.10}{3/200}\right)$

$= P(-0.67 < Z < 0.67)$

$= P(0.67) - P(-0.67)$

$= 0.7486 - 0.2514$

$= 0.4972.$

Ex- Suppose that 90% of the university students pass calculus 101. In a sample of 200 students taking Calculus 101, what is the probability that the proportion of those who will pass is less than 85%?

Soln. $p = 0.90$, $1 - p = 0.10$, $n = 200$.

$\hat{p} \sim N(0.90, (0.90)(0.10)/200)$

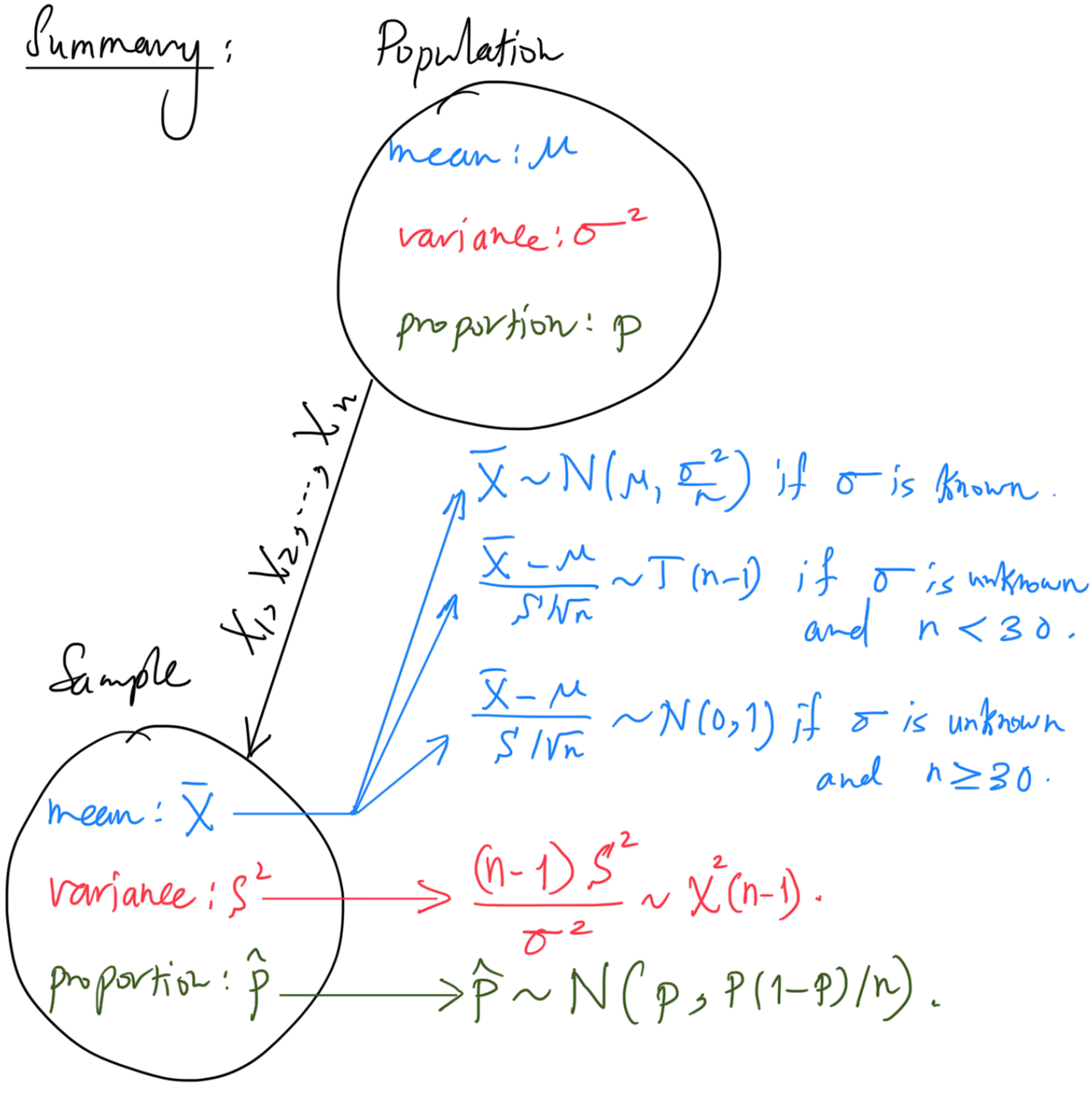
$$\hat{p} \sim N(0.90,\ 9/20000).$$

$$P(\hat{p} < 0.85) = P\left(\frac{\hat{p} - 0.90}{\sqrt{9/20000}} \leq \frac{0.85 - 0.90}{\sqrt{9/20000}}\right)$$

$$= P(Z < -2.36)$$

$$= 0.0091.$$

Summary:



Searching keywords:

- Sampling distributions, distribution of the sample proportion
- Find the probability of
- The University of Jordan الجامعة الأردنية
- Principles of Statistics مبادئ الإحصاء
- Baha Alzalg بهاء الزالق

References: See the textbook in the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/131.aspx

For any comments or concerns, please use my email to contact me.

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