Lecture #9

Specification of Sets

The set of all elements $x \in D$ such that the statement $P(x)$ is true is denoted by

$$\{x : D : P(x) \text{ is true}\}$$

and is called the set builder notation

Ex) $\{n \in \mathbb{N} : (n \geq 1) \wedge (n < 5)\} = \{1, 2, 3, 4\}$

use set builder notation to define

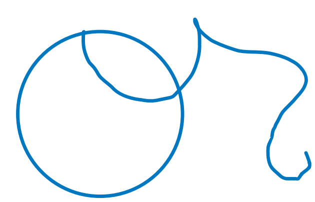
1. $\{-1, \sqrt{2}\}$

$$\{x \in \mathbb{R} : (x+1)(x-\sqrt{2}) = 0\}$$

2. $\emptyset \longrightarrow \{x \in \mathbb{N} \ (x^2 < 0)\}$

3. $(-3, 5]$

$$\{x \in \mathbb{R} \ -3 < x \leq 5\}$$



Conjunctive Normal Form (CNF)

- The conjunction of disjunctive clauses, & each clause is the disjunction of 1 or more literals

$\neg P$

$(P \vee \neg Q) \wedge \neg P \wedge (P \vee Q \vee R)$

$\neg Q \wedge R$

$P \wedge \neg Q$

(An ANDing of OR clauses

$\square \rightarrow$ CNF ?? $D \wedge D \wedge \ldots \wedge D$

$\neg \square \rightarrow$ DNF : $C \vee C \vee \ldots \vee C$

$\neg\neg\square \equiv \neg(C \vee C \vee \ldots \vee C)$

$\equiv \neg C \wedge \neg C \wedge \ldots \wedge \neg C$

$D \wedge D \wedge \ldots \wedge D$

Fact: We convert a prop. into CNF by 3 steps:

① Create a truth table for the negation of the prop.

② Use the table to create an equivalent prop. in DNF

③ Negate the prop. from ② and apply DeMorgan's & Double negation to convert prop to CNF

EX) Convert $(P \vee Q) \wedge \neg R$ to CNF

P	Q	$P \wedge Q$	R	$\neg R$	$(P \wedge Q) \vee \neg R$	$\neg (P \wedge Q) \vee \neg R$	
T	T	T	T	F	T	F	
T	T	T	F	T	T	F	
T	F	F	T	F	F	T	$\rightarrow P \wedge \neg Q \wedge R$
T	F	F	F	T	T	F	
F	T	F	T	F	F	T	$\rightarrow \neg P \wedge Q \wedge R$
F	T	F	F	T	T	F	
F	F	F	T	F	F	T	$\rightarrow \neg P \wedge \neg Q \wedge R$
F	F	F	F	T	T	F	

DNF

$\equiv (P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

$\equiv$

Satisfiable Proposition

Evaluates to true for some values

ex) $(P \vee Q) \rightarrow \neg P$

Satisfiable if P is F & Q T

ex) $(\neg P \wedge Q) \vee (P \wedge \neg Q \wedge R) \vee (\neg P \wedge \neg R)$

In DNF, it is enough to evaluate only 1 clause to true