Lecture #8

INTRO TO SET THEORY

Set Theory

A set is an ordered collection of distinct objects.

ex) vowels = {a, e, i, o, u}
days = {1, 2, 3... 365} - Finite
natural numbers = {1, 2, 3 ... } - Infinite

① The order is not significant
② No duplicates

Elements/ Members

The objects of a set

- Let S be a set

NOTATION : $x \in S$ = x is an element of S

ex) Saturday ∈ Weekend
1 ∈ Days - True
a ∈ Vowels
-4 ∈ Days - FALSE
Monday ∈ Weekend

Set Equality

Let A & B be two arbitrary sets,
A = B is true when A & B have precisely the same elements

ex) $\{1,2,3\} = \{3,1,2\}$

$\mathbb{N} \neq \mathbb{Z}$

The Empty Set ($\emptyset$ or $\{\ \}$)

Special set that has no elements

Subsets

$A \subseteq B$ is true when <u>all</u> elements of A are in B

ex) $\{1,2,3\} \subseteq$ Days

Proper Subset

$A \subset B$ is true when $A \subseteq B$ and $A \neq B$

ex) $\{1,2\} \subset \{1,2,3$

Power Set ($P(A)$)

A set of all subsets of A

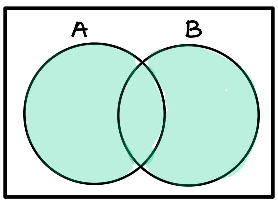
ex) $P(\{1\}) = \{\emptyset, \{1\}\}$

$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

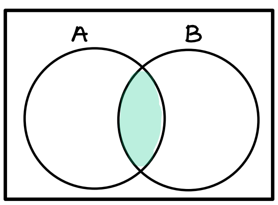
$P(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,2,3\}\}$
$\{2,3\}, \{1,3\}$

Operations on Sets

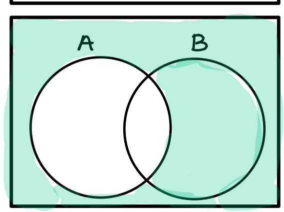
Venn Diagrams



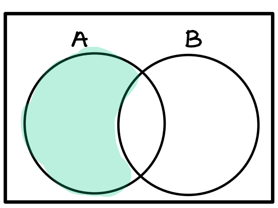
$A \cup B$



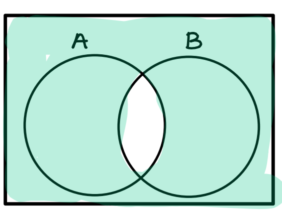
$A \cap B$



A^c (complement of A)

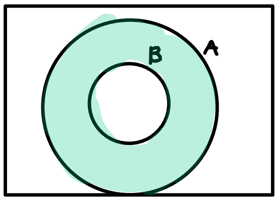


$A \cap B^c$

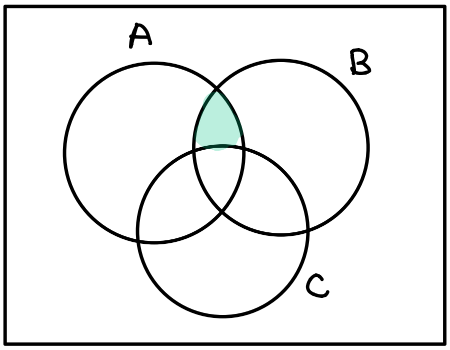


$(A \cap B)^c = A^c \cup B^c$

DeMorgan's Law



$A - 1$



$A \cap B \cap C^c$

① The union of A & B ($A \cup B$) contains both elements of A & B

② The intersection of A and B ($A \cap B$) contains only the common elements of both A & B

Cardinality of a Set

- # of elements in a set

ex) |weekend| = 2
(Saturday & Sunday)

$|\{1,2,3\}| = 3$

A = 1, 2, 3

B = 2, 3, 4

Union = {1, 2, 3, 4}

Intersection = {2, 3}