Lecture #7

DNF

Literal

A literal is an atomic prop. of the negation of an atomic proposition

$P, Q, \neg Q$ are literals

$P \wedge Q, P \vee Q \rightarrow$ NOT LITERALS

Conjunctive Clause

The conjunction of one or more literals

Disjunctive Normal Form

HELLO

– Clauses connected by OR

A prop is in DNF if it is the conjunction of one or more conjuctive clauses

$(P \wedge Q) \vee \neg R$, $P \wedge Q \rightarrow$ DNF

$\neg (P \wedge Q)$, $(P \wedge Q) \vee (R \rightarrow S) \rightarrow$ NOT DNF → NOT a clause

"An ORing of AND clauses"

Boolean functions

- TRUE or FALSE
- Named after George Boole

P	Q	R	f(P,Q,R)	
T	T	T	F	
T	T	F	T	→ $P \wedge Q \wedge \neg R$
T	F	T	F	
T	F	F	T	→ $P \wedge \neg Q \wedge \neg R$
F	T	T	F	
F	T	F	F	
F	F	T	T	→ $\neg P \wedge \neg Q \wedge R$
F	F	F	F	

Deriving a prop. formula for BOOLEAN func. from Truth Table

1) Write a conjunctive clause for each row where value of f is true

2) Write the disjunction of the conjunctive clauses obtained from step (1)

$$f(P,Q,R) = (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R)$$

Solve.

① $(P \wedge Q) \vee \neg R$

P	Q	R	$\neg R$	$(P \wedge Q)$	$(P \wedge Q) \vee \neg R$
T	T	T	F	T	(T) → $P \wedge Q \wedge R$
T	T	F	T	T	(T) → $P \wedge Q \wedge \neg R$
T	F	T	F	F	F
T	F	F	T	F	(T) → $P \wedge \neg Q \wedge \neg R$
F	T	T	F	F	F
F	T	F	T	F	(T) → $\neg P \wedge Q \wedge \neg R$
F	F	T	F	F	F
F	F	F	T	F	(T) → $\neg P \wedge \neg Q \wedge \neg R$

$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge Q \wedge \neg R) \wedge (\neg P \wedge \neg Q \wedge \neg R)$

② The negation of $P \rightarrow Q$

$\neg(P \rightarrow Q)$

Converting into DNF

1) convert symbols to $\wedge, \vee, \neg$ using logical equivalences
2) Use double negation and DeMorgan's so negations are directly on variables
3) Use commutative, distributive & associative laws as needed

ex) convert $(P \to (Q \to R)) \vee \neg(P \vee \neg(R \vee S))$

Implication

$\equiv (P \to (\neg Q \vee R)) \vee \neg(P \vee \neg(R \vee S))$

$\equiv (\neg P$

Homework:

https://u.osu.edu/alzalg.1/files/2019/09/hw3.pdf