Lecture # 6

Modeling in Prop. Logic

- Representation of English statement using variables & operators

ex) If today is Wed, and today is not a holiday, then we have class

P: "Today is Wed"
Q: "Today is a holiday"
R: "We have class"

MODEL: \( P \land \neg Q \rightarrow R \)

ex) If it is raining and you do not have an umbrella then you will get wet.

CONTRAPOSTIVE:

If you did not get wet
it is not raining and you have an umbrella
Using Truth Table to derive Prop. Formulas

\begin{align*}
\text{ex) } & \quad P \lor (\neg P \land Q) \equiv P \lor Q \\
\begin{array}{|c|c|c|c|c|c|}
\hline
P & \neg P & Q & (\neg P \land Q) & P \lor (\neg P \land Q) & P \lor Q \\
\hline
T & F & T & F & T & T \\
F & T & T & T & T & T \\
T & F & F & F & F & T \\
F & T & F & F & F & T \\
\hline
\end{array}
\end{align*}

\begin{align*}
\text{ex) } & \quad P \oplus (Q \land R) \equiv (P \oplus Q) \land (P \oplus R)
\end{align*}
### Important Laws in Prop. Logic

<table>
<thead>
<tr>
<th>Name</th>
<th>Law</th>
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</thead>
<tbody>
<tr>
<td><strong>1. Identity Laws</strong></td>
<td>$T \land P \equiv P$</td>
</tr>
<tr>
<td></td>
<td>$F \lor P \equiv P$</td>
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<tr>
<td><strong>2. Domination Laws</strong></td>
<td>$T \lor P \equiv T$</td>
</tr>
<tr>
<td></td>
<td>$F \land P \equiv F$</td>
</tr>
<tr>
<td><strong>3. Idempotence Law</strong></td>
<td>$P \lor P \equiv P$</td>
</tr>
<tr>
<td></td>
<td>$P \land P \equiv P$</td>
</tr>
<tr>
<td><strong>4. Tautology Law</strong></td>
<td>$P \lor \neg P \equiv T$</td>
</tr>
<tr>
<td><strong>5. Contradiction Law</strong></td>
<td>$P \land \neg P \equiv F$</td>
</tr>
<tr>
<td><strong>6. Implication Law</strong></td>
<td>$P \rightarrow Q \equiv \neg P \lor Q$</td>
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<tr>
<td><strong>7. Contrapositive Law</strong></td>
<td>$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$</td>
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<tr>
<td><strong>8. De Morgan’s Laws</strong></td>
<td>$\neg (P \land Q) \equiv \neg P \lor \neg Q$</td>
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<tr>
<td></td>
<td>$\neg (P \lor Q) \equiv \neg P \land \neg Q$</td>
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<tr>
<td><strong>9. Commutative Laws</strong></td>
<td>$P \land Q \equiv Q \land P$</td>
</tr>
<tr>
<td></td>
<td>$P \lor Q \equiv Q \lor P$</td>
</tr>
<tr>
<td></td>
<td>$P \oplus Q \equiv Q \oplus P$</td>
</tr>
</tbody>
</table>
| 10. Associative Laws | \( P \land (Q \land R) = (P \land Q) \land R \)  
|                     | \( P \lor (Q \lor R) = (P \lor Q) \lor R \)  
|                     | \( P \oplus (Q \oplus R) = (P \oplus Q) \oplus R \) |
| 11. Distributive Laws | \( P \land (Q \lor R) = (P \land Q) \lor (P \land R) \)  
|                     | \( P \lor (Q \land R) = (P \lor Q) \land (P \lor R) \)  
|                     | \( P \land (Q \oplus R) = (P \land Q) \oplus (P \land R) \) |
| 12. Exclusive-Or Laws | \( P \oplus Q = (P \lor Q) \land \neg (P \land Q) \)  
|                     | \( P \oplus Q = (P \land \neg Q) \lor (Q \land \neg P) \) |
| 13. Double Negation Laws | \( \neg \neg P \equiv P \) |
Using logical operators to derive propositional formulas

ex) \[ PV(\neg P \land Q) \equiv PVQ \]

\[ PV(\neg P \land Q) \equiv (PV\neg P) \land (PVQ) \]

\[ \equiv T \land (PVQ) \]

\[ \equiv PVQ \]

ex) \[ P \land Q \rightarrow P \equiv T \] (via a tautology)

\[ \neg (P \land Q) \lor P \]

\[ \neg (P \land Q) \lor P \]

\[ \neg (\neg Q \lor \neg P) \lor P \]

\[ \neg Q \lor (\neg P \lor P) \]

\[ \neg Q \lor T \]

\[ T \equiv T \]
Propositional Logic in CS

Java Code:

```
if (x<0 || (x>=0 && x%60, y==43))
    P || !P && Q

In P v (!P && Q)
```

Homework: