Lecture # 6

Modeling in Prop. Logic

- Representation of English statement using variables & operators

ex) If today is wed, and today is not a holiday, then we have class

P : "Today is Wed"
Q : "Today is a holiday"
R : "We have class"

MODEL: $P \land \neg Q \to R$

ex) If it is raining and you do not have an umbrella then you will get wet.

CONTRAPOSITIVE:

If you did not get wet
it is not raining and you have a umbrella

Using Truth Table to derive Prop. Formulas

ex) $P \vee (\neg P \wedge Q) \equiv P \vee Q$

P	$\neg P$	Q	$(\neg P \wedge Q)$	$P \vee (\neg P \wedge Q)$	$P \vee Q$
T	F	T	F	T	T
F	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	F	F

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ex) $P \oplus (Q \wedge R) \equiv (P \oplus Q) \wedge (P \oplus R)$

Important Laws in Prop. Logic

Name	Law
1. Identity Laws	$T \wedge P \equiv P$ $F \vee P \equiv P$
2. Domination Laws	$T \vee P \equiv T$ $F \wedge P \equiv F$
3. Idempotence Law	$P \vee P \equiv P$ $P \wedge P \equiv P$
4. Tautology Law	$P \vee \neg P \equiv T$
5. Contradiction Law	$P \wedge \neg P \equiv F$
6. Implication Law	$P \rightarrow Q \equiv \neg P \vee Q$
7. Contrapositive Law	$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$
8. DeMorgan's Laws	$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
9. Commutative Laws	$P \wedge Q \equiv Q \wedge P$ $P \vee Q \equiv Q \vee P$ $P \oplus Q \equiv Q \oplus P$

10. Associative Laws	$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$ $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ $P \oplus (Q \oplus R) \equiv (P \oplus Q) \oplus R$
11. Distributive Laws	$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$ $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \wedge R)$ $P \wedge (Q \oplus R) = (P \wedge Q) \oplus (P \wedge R)$
12. Exclusive-Or Laws	$P \oplus Q = (P \vee Q) \wedge \neg (P \wedge Q)$ $P \oplus Q = (P \wedge \neg Q) \vee (Q \wedge \neg P)$
13. Double Negation Laws	$\neg\neg P \equiv P$

Using Logical Operators to derive Prop. Formulas

ex) $P \vee (\neg P \wedge Q) \equiv P \vee Q$

$\boxed{P \vee} (\neg P \wedge Q) \overset{\text{Dist.}}{\equiv} \underbrace{(P \vee \neg P)}_{\text{Taut.}} \wedge (P \vee Q)$

$\equiv T \wedge (P \vee Q)$

Identity

$\equiv P \vee Q$

ex) $P \wedge Q \rightarrow P \equiv T$ (is a tautology)

Implication

$\neg(P \wedge Q) \vee P$

DeMorgan's

$(\neg P \vee \neg Q) \vee P$

Commutative

$(\neg Q \vee \neg P) \vee P$

Associative

$\neg Q \vee (\neg P \vee P)$

Tautology

$\neg Q \vee T$

Domination

$T \equiv T$

Propositional Logic in C.S.

Java Code:

$$\text{if } (\underbrace{x<0}_{P}\ \underbrace{||}_{\text{OR}}\ (\underbrace{x>=0}_{\neg P}\ \underbrace{\&\&}_{\text{AND}}\ \underbrace{y==43}_{Q}))$$

In $\quad P \vee (\neg P \wedge Q)$

Homework:

https://u.osu.edu/alzalg.1/files/2019/08/HW2.pdf