

# Lecture 5

## TRUTH TABLES

-  $(P \wedge Q) \vee \neg R$

Truth Table:

P	Q	R	$\neg R$	$P \wedge Q$	$(P \wedge Q) \vee \neg R$
T	T	T	F	T	T
F	T	T	F	F	F
T	F	T	F	F	F
F	F	T	F	F	F
T	T	F	T	T	T
F	T	F	T	F	T
T	F	F	T	F	T
F	F	F	T	F	T

### Number of Rows in a Truth Table

If we have  $n$  variables, we have  $2^n$  rows

### Order of Operations

- Like PEMDAS in arithmetic
- For propositional logic

1)  $\neg$  Not    2)  $\wedge$  And    3)  $\vee$ ,  $\oplus$  Or, XOR    4)  $\rightarrow$  Implication    5)  $\leftrightarrow$  IFF

EX) Find the value of prop. formula

$$\begin{aligned} & P \vee Q \wedge R \quad P=F \quad R=F \\ & \quad Q=T \\ & \rightarrow P \vee (Q \wedge R) \\ & \quad F \vee (T \wedge F) \\ & \quad F \vee (F) \\ & \quad = \text{False} \end{aligned}$$

EX) Use parentheses to group expressions

$$\begin{aligned} & P \vee Q \wedge R \rightarrow \neg R \vee Q \\ & (P \vee (Q \wedge R)) \rightarrow ((\neg R) \vee Q) \\ & F \vee (T \wedge F) \rightarrow (T \vee T) \\ & F \vee F \rightarrow T \\ & \quad F \rightarrow T \\ & \quad \text{True} \end{aligned}$$

## Tautologies, Contradictions & Contingencies

Tautology - A prop. formula that is always true.

ex)  $P \vee \neg P$

Truth Table

P	$\neg P$	$P \vee \neg P$
T	F	T
F	T	T

Equivalence - If  $P \leftrightarrow Q$  is a tautology.

ex)  $(P \rightarrow Q) \equiv (\neg P \vee Q)$

Contradiction - A prop. formula that is always false.

ex)  $P \wedge \neg P$

Truth Table

P	$\neg P$	$P \wedge \neg P$
T	F	F
F	T	F

Contingency - Not a tautology or contradiction

## Negating compound propositions

DNL = Double Negation Law

$$\neg\neg P \equiv P$$

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what about  $\neg(P \wedge Q)$   
 $\neg(P \vee Q)$   
 $\neg(P \rightarrow Q)$ ?

Example:

"I will get an A in 2321 & 1172"

IF FALSE)

↳ "I will get an A in neither"

## DeMorgan's Law

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$\neg P \vee \neg Q$
T	T	F	F	T	F	F
F	T	T	F	F	T	T
T	F	F	T	F	T	T
F	F	T	T	F	T	T

## Second DeMorgan's Law

$$\neg(P \vee Q) = P \wedge \neg Q$$

1)  $P \wedge Q \vee \neg R$

2)  $P \wedge Q \wedge (R \vee \neg P)$

3)  $P \leftrightarrow Q$

4)  $P \oplus Q$