Lecture #4

Why Implications are Important:

- many Math theorems can be stated in IF → THEN form

EX: Prove that the sum of two even integers is even.

IF x and y are even integers
THEN x + y is an even integer

PROOF: Let x & y be even

Want: $x+y$ is even

$$\left.\begin{aligned} x &= 2k_1 \\ y &= 2k_2 \end{aligned}\right\} \text{for some } k_1, k_2 \in \mathbb{Z}$$

$$\begin{aligned} x+y &= 2k_1 + 2k_2 \\ &= 2\underbrace{(k_1 + k_2)}_{k_3} \\ &= 2k_3 \end{aligned}$$

RECALL:
The sum of two integers is an integer

x is odd, $x = 2k+1$

x is even $x = 2k$

FOR SOME k

EX: Prove the sum of two odd integers is even

Contrapositive

$$P \to Q$$
$$\neg Q \to \neg P$$

Negation of conclusion leads to negation of hypothesis

ex) P: "Today is Wednesday"
Q: "We have a class today"

$P \to Q$: If today is Wed, we have a class today

$\neg P \to \neg Q$: If we don't have a class today today is not wednesday

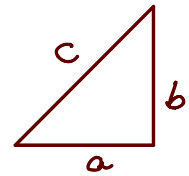
(Truth Table:

P	Q	$P \to Q$	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	T	F	F	T
F	T	T	F	T	T
T	F	F	T	F	F
F	F	T	T	T	T

FACT: $P \to Q \equiv \neg Q \to \neg P$ (logically equivalent)

Contrapositive - once a theorem is proven, there is no need to prove its contrapositive

Ex: Pythagorean theorem says
IF a triangle has a right angle
THEN $a^2+b^2=c^2$



Contrapositive: If $a^2+b^2 \neq c^2$,
Then the triangle does not have a right angle

* Sometimes it's easier to prove the contrapositive of a theorem.

Ex Let x be an integer prove that if x^2 is even then x is even

prove contrapositive: If x is odd then x^2 is odd

x is odd, so $x = 2k+1$

$$x^2 = (2k+1)^2$$
$$(2k+1)(2k+1)$$
$$4k^2+2k+2k+1$$
$$4k^2+4k+1$$
$$2\underbrace{(2k^2+2k)}+1$$
$$2 * \text{something} + 1$$

ODD

Converse

$P \to Q$

$Q \to P$

ex) P: "Today is Wednesday" × × ×

Q: "We have a class today"

$Q \to P$ IF we have a class today THEN today is Wednesday (FALSE

Inverse

$P \to Q$

$\neg P \to \neg Q$

ex) P: "Today is Wednesday"

Q: "We have a class today"

$\neg P \to \neg Q$ IF today is not wednesday THEN we don't have a class today

(FALSE

(Truth Table:

P	Q	$P \to Q$	$Q \to P$	$\neg P$	$\neg Q$	$\neg P \to \neg Q$
T	T	T	T	F	F	T
F	T	T	F	T	F	F
T	F	F	T	F	T	T
F	F	T	T	T	T	T

$Q \to P \equiv \neg P \to \neg Q$

$\neg P \to \neg Q$ is the contrapositive of $P \to Q$

Biconditional – IF AND ONLY IF

$P \leftrightarrow Q$

goes both ways

(Truth Table:

P	Q	$P \leftrightarrow Q$
T	T	T
F	T	F
T	F	F
F	F	T