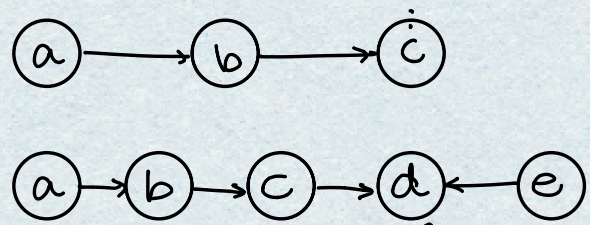
Lecture 33

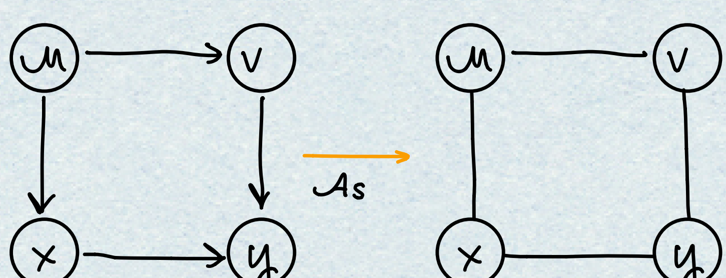
Directed Graphs

Path (in a digraph)
a sequence of distinct verticies such that there exists a directed edge from each vertex to the next vertex on the page



Cannot get from a to e

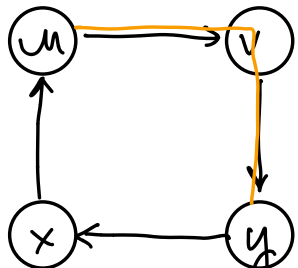
Weakly Connected – replacing all directed w/ undirected edges produces a connected (undirected) graph



Weakly Connected

This undirected graph is connected

Strongly Connected - every 2 vertices are reachable from each other



You can go from u to y, y to u, v to x...

There is a direct path from every vertex to another

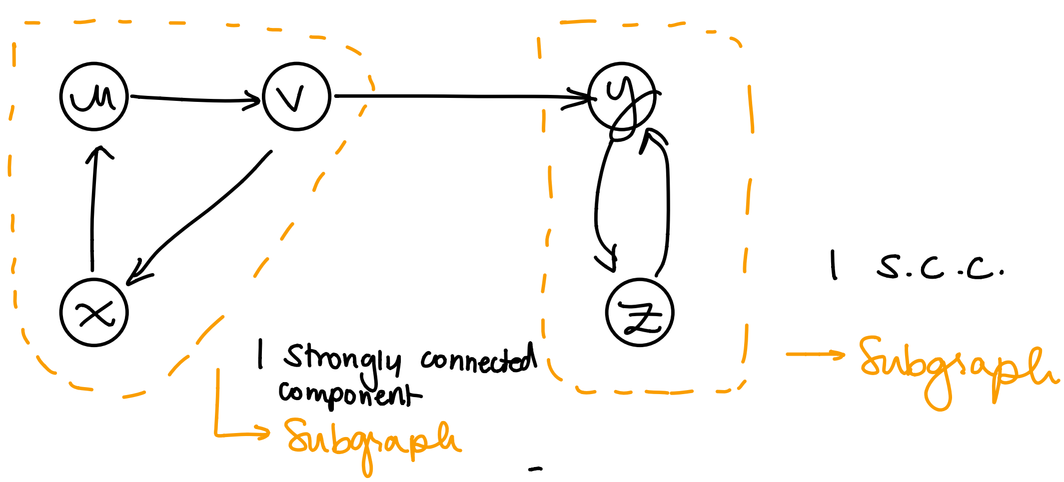
(ALSO weakly connected)

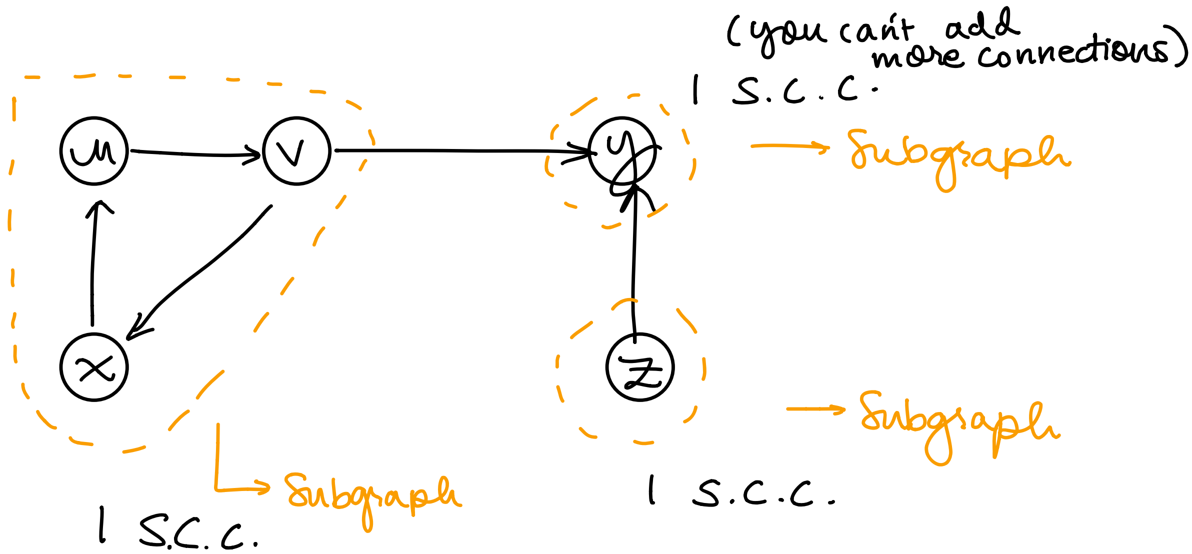
* The whole graph should only have 1 S.C.C.

Strongly Connected Components (S.C.C)

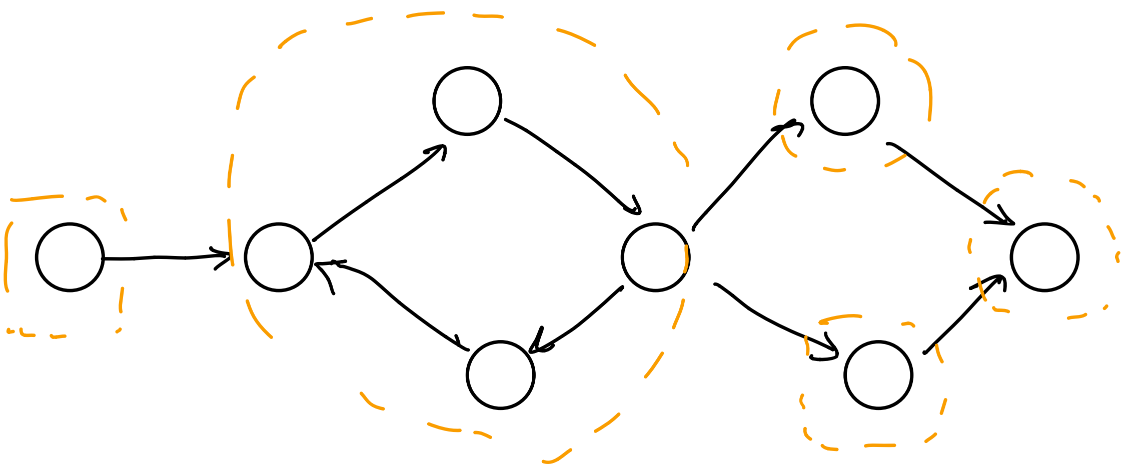
G' of a digraph G is a maximal strongly connected subgraph of G.

Ex.





Example (like Q #2 HW #14)



5 strongly connected components

Graph Representation

Motivation - present algorithms for searching & exploring a graph

2 Standard Ways to Represent Graphs

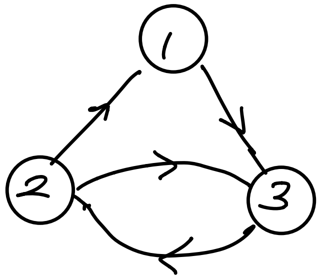
① Adjacency List Representation of $G = (V, E)$

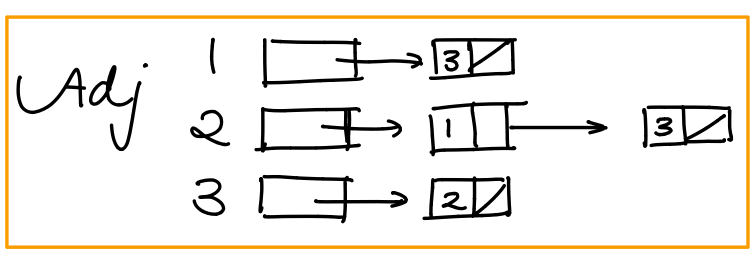
- An array of $|V|$ lists
  - one list for each vertex in V
- Each list $Adj[u]$ contains all the verticies v that are adjacent to u.

$$Adj[u] = \{ v \in V : (u, v) \in E \}$$

Ex. use adjacency lists to rep. the graphs

Directed



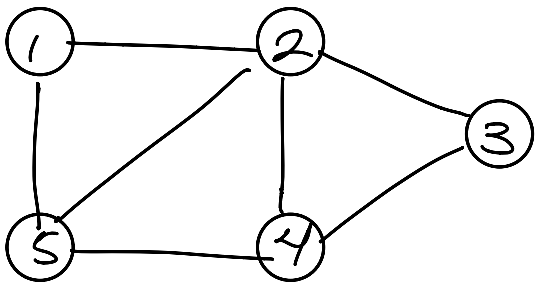


$Adj[1] = \{3\}$

$Adj[2] = \{1, 3\}$

Adj [3] = {2}

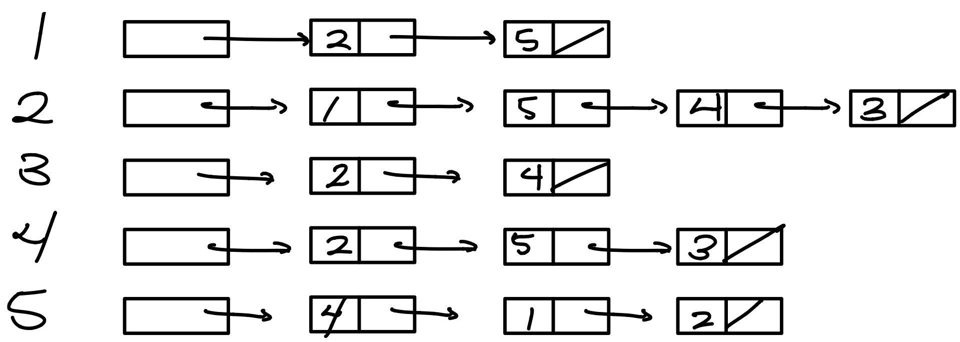
Undirected



Adj[1] = {2, 5}
Adj[2] = {1, 5, 4, 3}
Adj[3] = {2, 4}
Adj[4] = {2, 5, 3}
Adj[5] = {4, 1, 2}

Q #3, Hw #14

Adj



② Adjacency Matrix Representation

Idea: Assume verticies are numbers

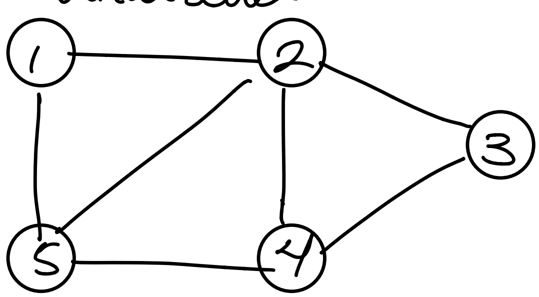
$1, 2, 3, \ldots, |V|$

– representation consists of a matrix

$A \; |V| \times |V|$:

$$a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$$

Undirected



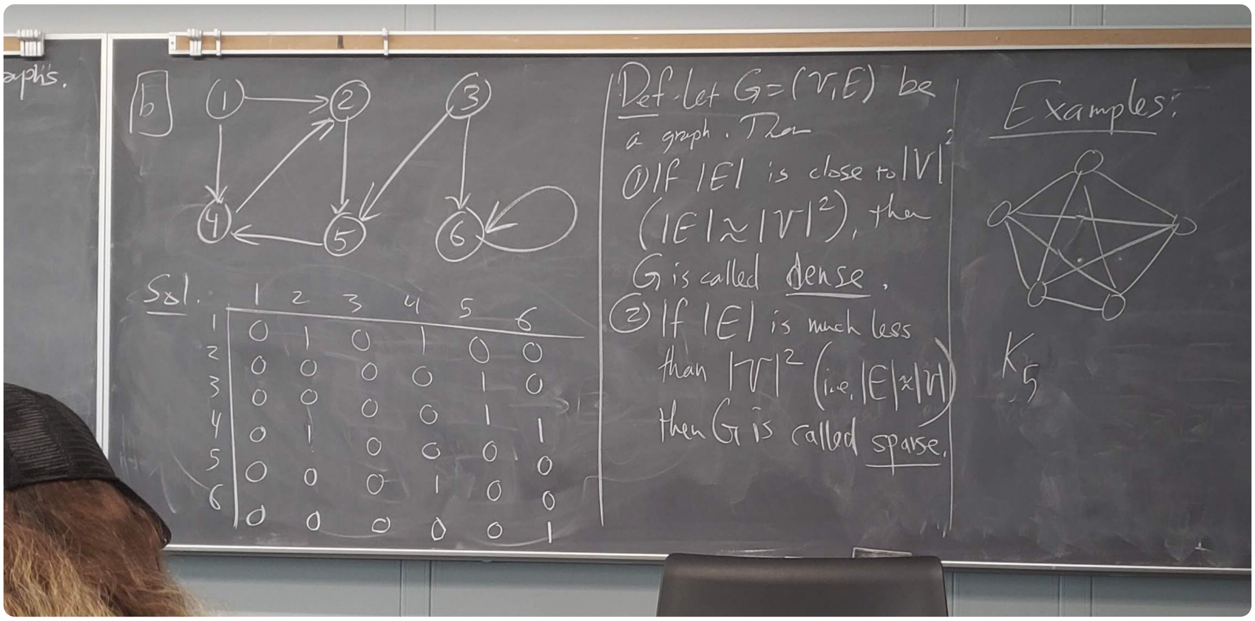
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 1 & 0 & 1 \\ 5 & 1 & 1 & 0 & 1 & 0 \end{array}$$

$A = A^T$

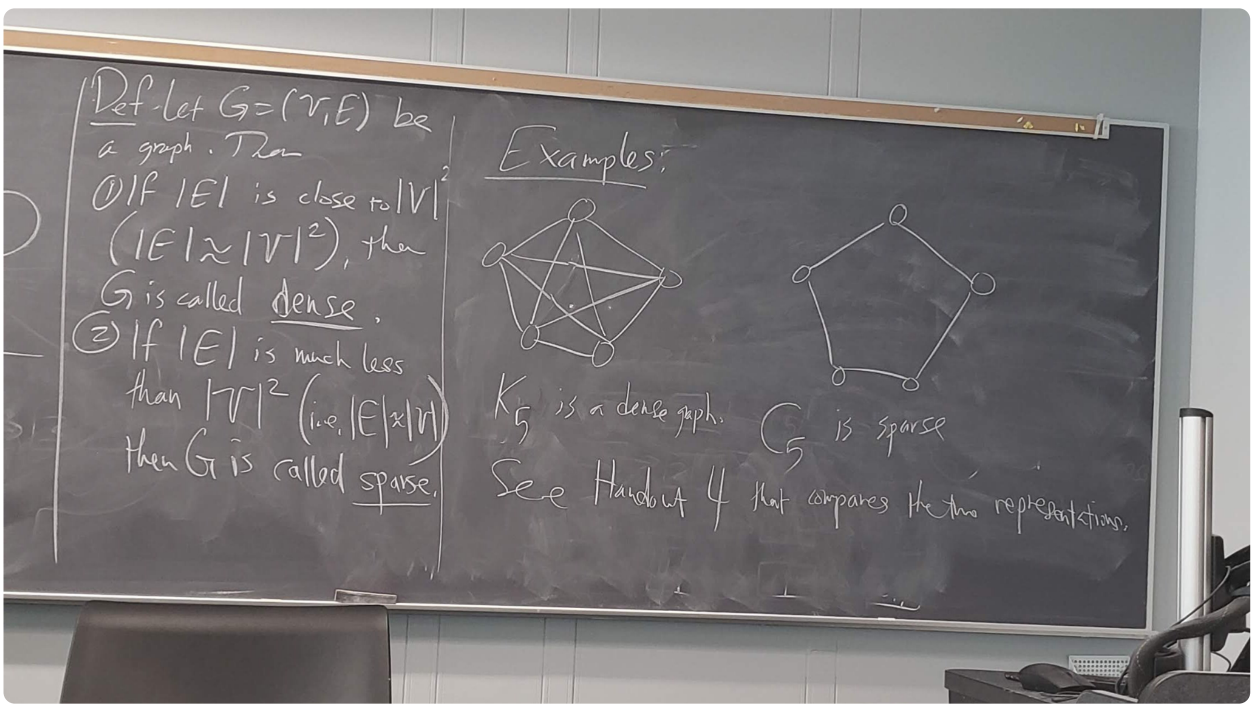
symmetric

no self-loops

Directed



Sol.
Def. Let G=(V,E) be a graph. Then
① If |E| is close to |V|^2 (|E| ≈ |V|^2), then G is called dense.
② If |E| is much less than |V|^2 (i.e. |E| ≈ |V|) then G is called sparse.
Examples:
K_5



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Examples:
K_5 is a dense graph.
C_5 is sparse
See Handout 4 that compares the two representations.