Directed Graphs

Path (in a digraph)
a sequence of distinct vertices
such that there exists a directed edge
from each vertex to the next vertex
on the page

\[ a \rightarrow b \rightarrow c \]
\[ a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \]

Cannot get from \( a \) to \( e \)

Weakly Connected - replacing all directed
w/ undirected edges produces a connected
(undirected) graph

\[ x \rightarrow y \]
\[ X \rightarrow Y \]

Weakly Connected

\[ M \rightarrow V \]
\[ M \rightarrow V \]

As

This undirected graph is connected
**Strongly Connected** - every 2 vertices are reachable from each other

You can go from u to y, y to u, v to x...

There is a direct path from every vertex to another

(ALSO weakly connected)

* The whole graph should only have 1 S.C.C.

**Strongly Connected Components (S.C.C.)**

$G'$ of a digraph G is a maximal strongly connected subgraph of G.

Ex.

1 Strongly connected component

2 Subgraphs
Example (like Q #2 HW #14)
Graph Representation

Motivation: present algorithms for searching & exploring a graph

2 Standard ways to Represent Graphs

(1) Adjacency List Representation of $G = (V, E)$

- An array of $|V|$ lists
- one list for each vertex in $V$
- Each list $\text{Adj}[u]$ contains all the vertices $v$ that are adjacent to $u$.

$\text{Adj}[u] = \{ v \in V : (u, v) \in E \}$

Ex: use adjacency lists to rep. the graphs

Directed

\[ \begin{align*}
\text{Adj} & \quad 1 \quad 2 \quad 3 \\
1 & \rightarrow \boxed{3} \\
2 & \rightarrow \boxed{1} \rightarrow \boxed{3} \\
3 & \rightarrow \boxed{2} \\
\end{align*} \]

$\text{Adj} [1] = \{3\}$
$\text{Adj} [2] = \{1, 3\}$
Adj [3] = \{2, 5\}
Adj [1] = \{2, 5\}
Adj [2] = \{1, 5, 4, 3\}
Adj [3] = \{2, 4\}
Adj [4] = \{2, 5, 3\}
Adj [5] = \{4, 1, 2\}

Q #3, HW #14

Adj

1 2 5

2 1 5 4 3

3 2 4

4 2 5 3

5 4 1 2
2. **Adjacency Matrix Representation**

Idea: Assume vertices are numbers

\[ 1, 2, 3, \ldots, |V| \]

- Representation consists of a matrix

\[ A_{|V| \times |V|} : \]

\[ a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases} \]

**Undirected**

\[ \begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 \\
1 & 0 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 1 & 1 \\
4 & 0 & 1 & 0 & 1 \\
5 & 1 & 0 & 1 & 0 \\
\end{array} \]

\[ A = A^T \]

**Symmetric**

**No self-loops**
Directed

Def: Let $G = (V, E)$ be a graph. Then:
0. If $|E| \approx |V|^2$, then $G$ is called dense.
1. If $|E|\approx |V|^2$ (i.e., $|E| \ll |V|$) then $G$ is called sparse.

Examples:
- $K_5$ is a dense graph.
- $G$ is sparse.

See Exercise 4 that compares the two representations.