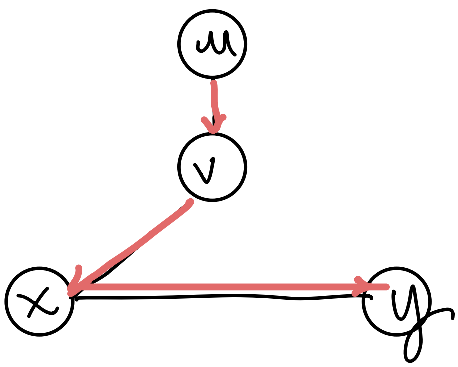
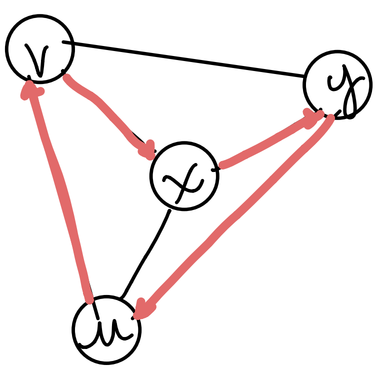
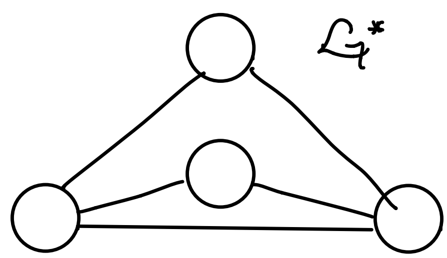
Lecture 32

Hamiltonian path: A path in a graph that visits each vertex exactly once



Hamiltonian Cylce - A Hamiltonian path that starts and ends on the same vertex





G^*

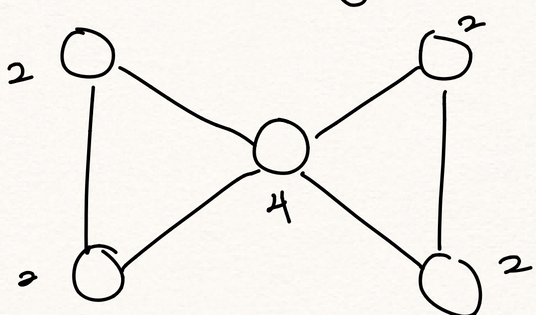
Dirac's Theorem - If each vertex of a connected graph w/ n verticies is adjacent to at least n/2 verticies, then the graph has an H.C.

In G^*, $\frac{n}{2} = 2 \le deg(v)$ for $v \in V$

So G^* has an H.C

The condition of Dirac's thm is sufficient (not necessary or sufficient)

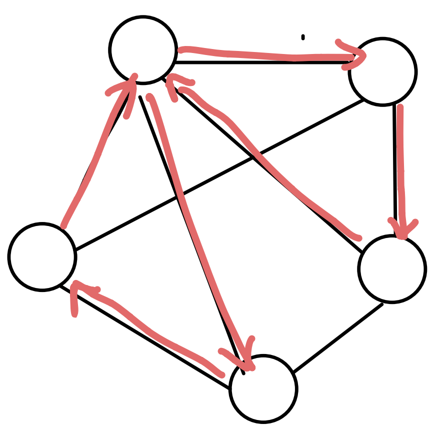
Take the <u>ceiling</u> if $n/2$ is odd



$\frac{5}{2} = \lfloor 2.5 \rfloor = 2$

$\lceil 2.5 \rceil = 3$

↳ NOT an H.C.



5 vertices

$\frac{n}{2} = 3$

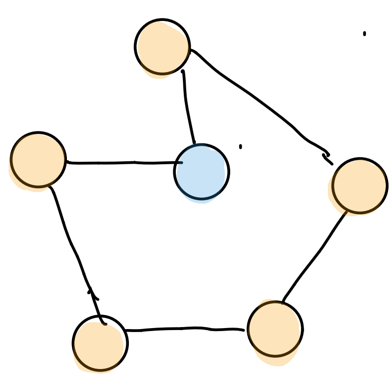
① A complete bipartite graph $K_{n,m}$ has an HC if $n = m$

② A complete bipartite graph $K_{n,m}$ has a HP if n and m are identical or differ by <u>one</u>

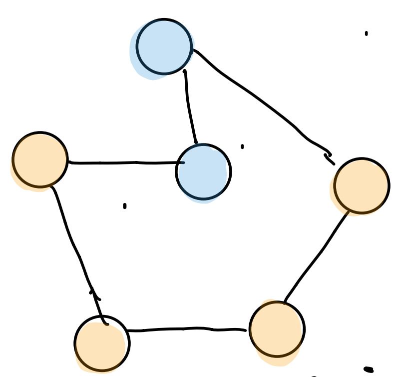
ex. $K_{5,7}$ has no HC nor HP

Graph Coloring

A proper coloring is an assignment of colors to the vertices of a graph so that no 2 adjacent vertices have the same color.



proper coloring.



Improper Coloring.

K-coloring of a graph is a proper coloring involving a total of K colors.

A graph of a K-coloring is called a K- colorable

Every graph $G=(V,E)$ is $|V|$-colorable.

* We are interested in the minimum # of colors

The complete graph K_n is <u>not</u> $(n-1)$-colorable

Pf. Consider any color assignment on the verticies of K_n that uses at most $(n-1)$ colors. Since there are n verticies, there exist two verticies u & v that share a color. However, since K_n is complete, (u,v) is an edge of the graph. This edge has 2 endpoints w/ the same color; so this scaling is improper. Thus, K_n is not $(n-1)$-colorable

Thm. Let G be a graph. Then G is 2-colorable IFF G is bipartite.

(Pf. in HW 14)

The Chromatic Number of a graph G, $\chi(G)$ is the minimum integer K such that G is K-colorable

Ex. $\chi(\text{Bipartite}) \leq 2$

Chi

$\chi(\text{Complete Bipartite}) = 2$

Fact: If H is a subgroup of a graph G, then $\chi(H) \leq \chi(G)$

Directed graphs (digraph)

a graph $G = (V, E)$ where (u, v) represents an edge from vertex u to vertex v.

Edge (u, v) is called a <u>directed edge</u> and represented by an arrow

$$\textcircled{u} \xrightarrow{\quad} \textcircled{v}$$

(u, v)

* we assume no self-loops.

<u>Edge (u, v)</u>

Incident from : u (incoming edge)

Incident to : v (outgoing edge)

* one way streets, Task scheduling

<u>in-degree</u> of vertex v: $d_{in}(v)$

Defs.

① The in-degree of a vertex v, denoted $d_{in}(v)$, is the number of edges incident to v:

$$d_{in}(v) = \left|\{(u,v) \in E : u \in V\}\right|.$$

② The out-degree of a vertex v, denoted by $d_{out}(v)$, is the number of edges incident from v:

$$d_{out}(v) = \left|\{(v,u) \in E : u \in V\}\right|.$$

Ex. $u \longrightarrow v$

$d_{in}(u) = 0$ $\quad$ $d_{in}(v) = 1$

$d_{out}(u) = 1$ $\quad$ $d_{out}(v) = 0$

Fact:

$$\sum_{v \in V} d_{in}(v) = \sum_{v} d_{out}(v) = |E| = m$$

$a \longrightarrow b \longrightarrow c$

$d_{in}(a) = 0$ $\quad$ $d_{in}(b) = 1$ $\quad$ $d_{in}(c) = 1$

$d_{out}(a) = 1$ $\quad$ $d_{out}(b) = 1$ $\quad$ $d_{out}(c) = 0$