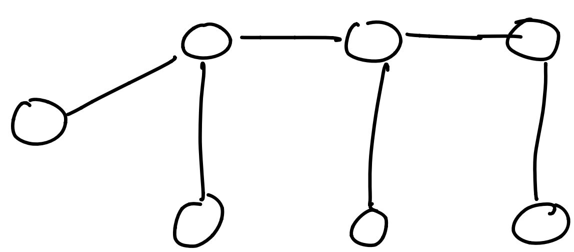
Lecture 30

<u>Tree</u> - connected undirected graph w/ no cycles

<u>Forest</u> - any undirected graph w/out cycles



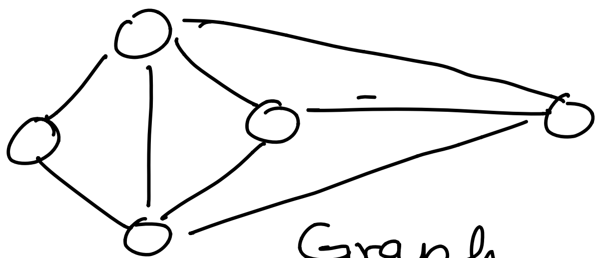
- All trees are forests
- Not all forests are trees

tree & forest

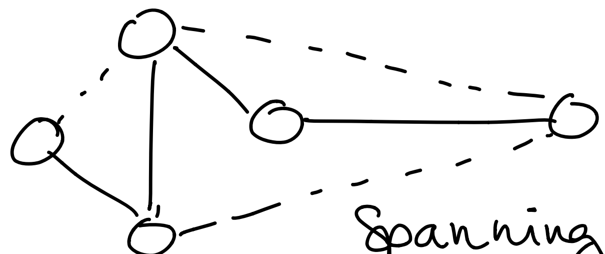
* connected components of a forest are trees

<u>Spanning Tree</u> of a connected graph is a spanning subgraph that is a tree

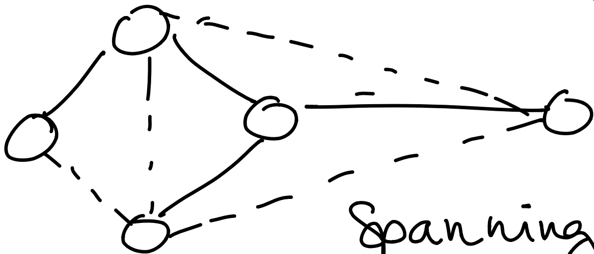
- ST is not unique unless the graph is a tree
- application to design of communication networks



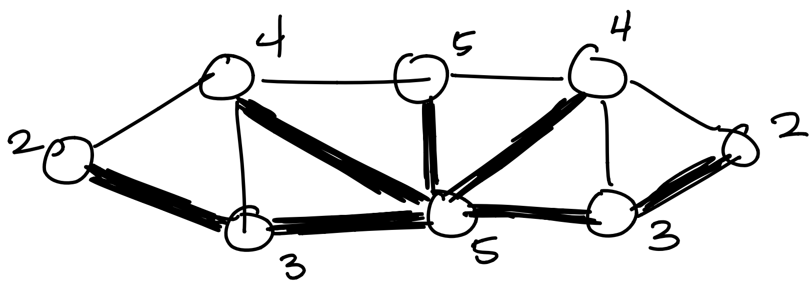
Graph



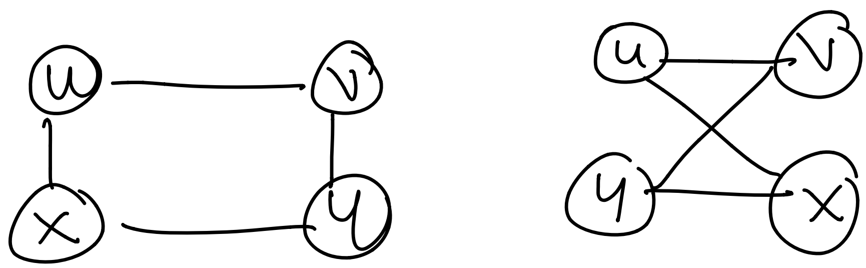
Spanning Tree



Spanning Tree



Two graphs are the same/isomorphic
- when the verticies of one can be relabeled to match the verticies of the other in a way that preserves adjacency



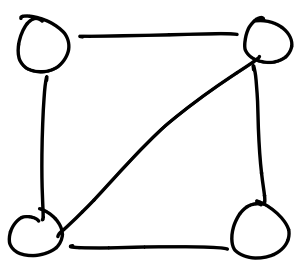
These graphs are isomorphic

A <u>spanning forest</u> of a graph is a spanning subgraph that is a forest.

- Unconnected graph - spanning forest
- Connected graph - spanning tree

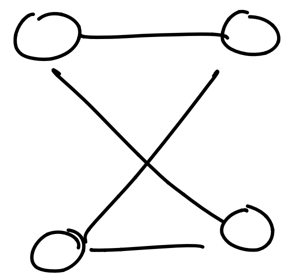
Identifying if graphs are NOT isomorphic

① If 2 graphs do not have the same # of edges & verticies, they are NOT isomorphic



$|V| = 4$

$|E| = 5$



$|V| = 4$

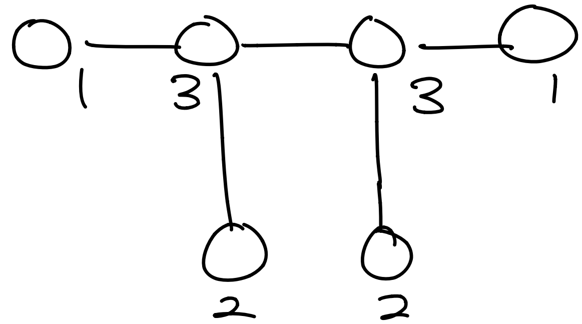
$|E| = 4$

Having the same # of V & E Does NOT guarantee the graphs are isomorphic

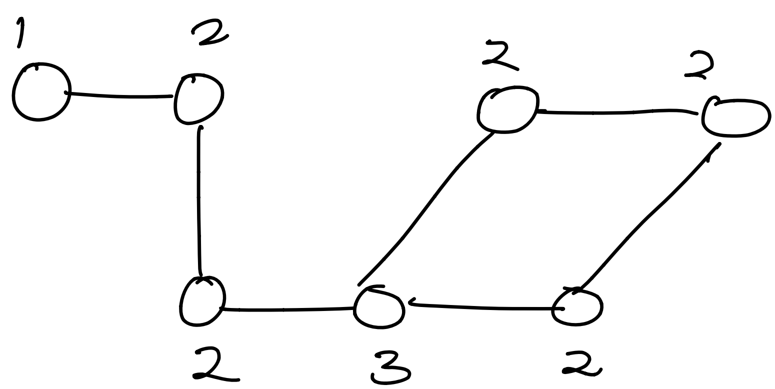
② The degree list of an undirected graph is the non-decreasing sequence of its vertex degrees

If 2 graphs have different degree lists then they are not isomorphic

Ex.



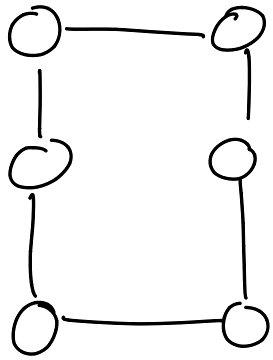
Degree List - 1, 1, 2, 2, 3, 3



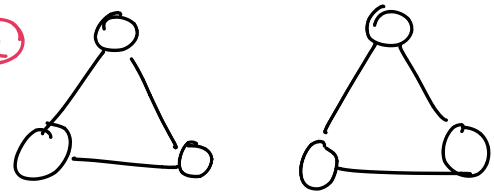
Degree List - 1, 2, 2, 2, 2, 3

Not isomorphic

1



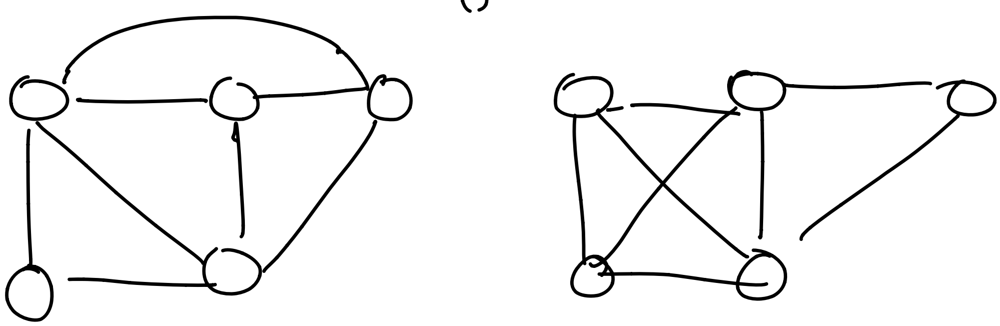
2



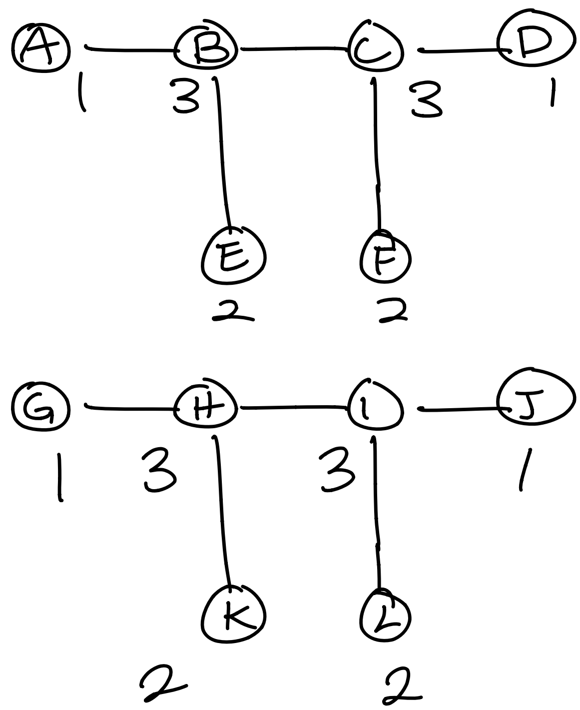
NOT isomorphic

same V, E & degree lists

Are these 2 graphs isomorphic?

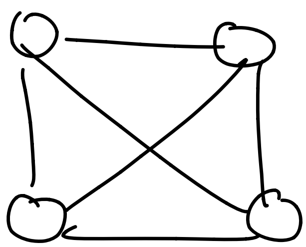


Test bonus.

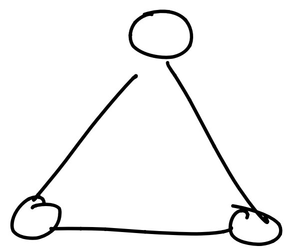


A <u>complete graph</u> (of n verticies), K_n, is a graph in which every pair of verticies is adjacent

Ex.



K_4



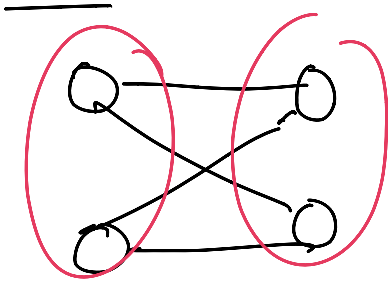
$K_3 = C_3$

<u>Bipartite graph of n verticies</u> is a graph in which the verticies can be partitioned into 2 sets A and B such That for every edge (u,v), u is in one of the sets and v is in The other.

Complete Bipartite Graph,

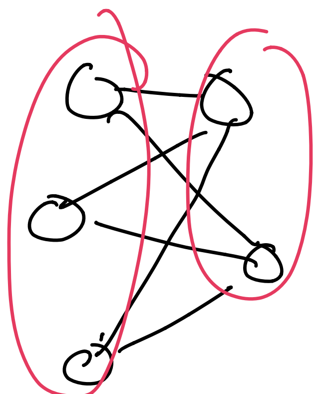
denoted by $K_{m,n}$ is a bipartite graph in which each ($|A| = m$, $|B| = n$) vertex in the first set is joined to each vertex in the 2ND

ex



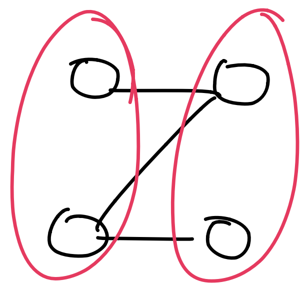
Complete Bipartite

$K_{2,2}$



Complete Bipartite

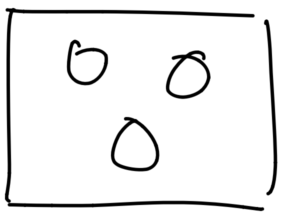
$K_{3,2}$



Bipartite (Not complete)

An <u>empty graph</u> is a graph w/ NO edges ($E = \emptyset$)

ex.

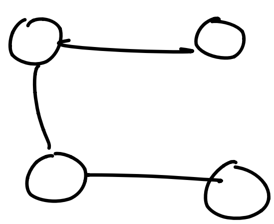


<u>Bridge / Cut - edge</u>

an edge of a graph whose deletion increases its # of connected components

<u>Ex</u>

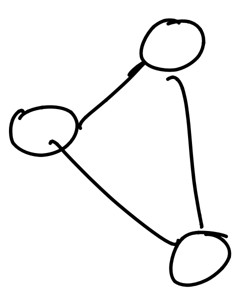
G_1 :



# of edges = 3

" " bridge edges = 3

G_2 :



edges = 3

bridge edges = 0

Sery Guna.

For undirected graph G, we assume

$n = |V|$ = # verticies of G

$m = |E|$ = # edges (cardinality)

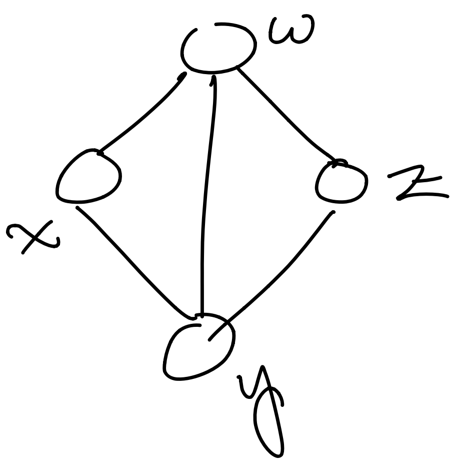
$\deg(u)$ = degree of vertex $u \in V$

Properties

Thm. In undirected graph G, we have

$$\sum \deg(u) = 2m$$

*each edge is connected to 2 verticies



$\deg(x) = 2$

$\deg(y) = 3$

$\deg(z) = 2$

$\deg(w) = 3$

$2+3+2+3 = 10 = 2m = 2(5)$

Thm. In an undirected graph, we have

$$m \leq \frac{n(n-1)}{2}$$

pf.

By thm, $m = 1/2 \sum deg(v)$

$$\leq \frac{1}{2} \sum_{v} (n-1)$$

$$deg(v) \leq n-1$$

$$m \leq \frac{1}{2}(n^2-n) = O(n^2)$$

That is $|E| = O(|V|^2)$