Lecture #3

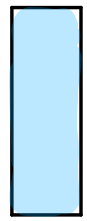
Implications

- Implication P implies Q (conditional of P and Q) (P: hypothesis, Q: conclusion)
  is written $P \to Q$ (if P then Q)
  - False when P is true & Q is false
  - True otherwise

ex) P: You live in America
Q: You live in the greatest nation in the world

IF you live in A, you live in the greatest nation

P	Q	$P \to Q$
T	T	T
F	T	F
T	F	F
F	F	T



ex) P: If everyone gets an A
Q: Alzalg will make cookies

* says nothing about what will happen if everyone does NOT get an A
↳ Q can be anything

All propositions are implications, so lines 1, 2, 4 are true and line 3 is false

$P \rightarrow Q$ equivalent forms

① If P then Q
② Q if P
③ P is sufficient for Q
④ Q is necessary for P
⑤ P only if Q

A) ① If a person is the president of the US (P), s/he is at least 35 years old (Q)

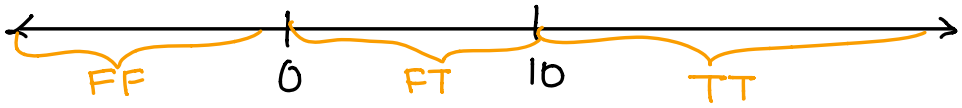
④ A person being at least 35 years old is necessary for being President
③ Being the President is sufficient for being at least 35
⑤ A person is The President only if they are at least 35

B) A passing score is required to receive a passing grade in the course

TF is always FALSE

ex) Let x be a real number.

A) If $x \geq 10$, then $x \geq 0$ TRUE



B) If $x \geq 0$, then $x \geq 10$ FALSE

C) If $x^2 \geq 0$, then $x = 42$ FALSE

D) If $x^2 < 0$, then $x = 42$ TRUE

P always false, so Q doesn't matter

Vacuous Truth

E) If Riemann hypothesis holds, then $x^2 \geq 0$ TRUE

always true, so P doesn't matter

Homework:

https://u.osu.edu/alzalg.1/files/2019/08/hw1-1.pdf