Lecture 29

Graph Theory
(continued)

TERMINOLOGY

- Two edges \( u \) and \( v \) are adjacent if there exists an edge \( (u, v) \).
- A self-loop is an edge \( (u, u) \).

If a graph does not have parallel edges & self loops it is **SIMPLE**.

A multigraph can have multiple edges between the same 2 vertices and self loops.

If \( (u, v) \) is an edge in graph \( G \), then \( (u, v) \) is INCIDENT to vertices \( u \) and \( v \).

- Degree of a vertex is the # of edges incident on it.
- A vertex whose degree is 0
A path of length $k$ from a vertex $u$ to a vertex $w$ is a sequence $(v_0, v_1, v_2)$ of vertices such that:

1. $u = v_0$
2. $w = v_1$
3. $(v_i, v_{i+1})$ is an edge in $E$ for $i = 1, 2, 3, \ldots, k$

A path is **simple** if all vertices in the path are distinct. Revisiting vertices & edges are allowed in **NON simple** paths.

- **Subpath** - of a path is a continuous subsequence of its vertices.

- **Cycle** - path in which first vertex = last vertex & all edges are distinct except first & last are distinct.
- A **cyclic** graph w/ no simple cycles

If $G = (V, E)$ is a graph, a graph $H = (V', E')$ is a subgraph of $G$ if $V' \subseteq V$ & $E' \subseteq E$

- A spanning subgraph of $G$ is a subgraph of $G$ that contains all vertices of $G$.

\[ H = V' \]
\[ E' \text{ is a spanning subgraph of } G \text{ if } V' \subseteq V \text{ and } E' \subseteq E \]

- A graph is connected if every vertex is reachable from all other vertices.

- A connected component $G'$ of graph $G$ is a minimal connected subgraph of $G$. 

has a path
MAXIMAL: there is no way to add into \( G' \) any vertices and/or edges of \( G \) which are not currently in \( G' \) in such a way that the resulting subgraph is connected.