Lecture 29

Graph Theory
(continued)

TERMINOLOGY

- 2 edges u & v are adjacent if there exists an edge of (u,v)
- A self-loop is an edge (u,u)

If a graph does not have parallel edges & self loops it is SIMPLE

- a multi-graph can have multiple edges b/w the same 2 verticies and self loops

If (u,v) is an edge in graph G, then (u,v) is INCIDENT to verticies u and v.

- Degree of a vertex is the # of edges incident on it.
A vertex whose degree is 0

is ISOLATED

- A path of length k from a vertex u to a vertex u' is a sequence (v0, v1, v2) of verticies such that

  u = v0

  u' = v1

  (v(i-0), vi) is an edge in E for i = 1, 2, 3, ... k

- A path is <u>SIMPLE</u> if all verticies in the path are distinct
  - Revisiting vertices & edges are allowed in <u>NON</u> simple paths

- <u>Subpath</u> - of a path is a continuous subsequence of its vertices

- <u>Cycle</u> - path in which first vertex = last vertex & all edges are distinct
  - Simple if all verticies except first & last are distinct

- Acyclic - graph w/ no simple cycles

If $G = (V, E)$ is a graph, a graph $H = (V', e')$ is a subgraph of G if $V' \leq V$ & $e' \leq e$

- A spanning subgraph of G is a subgraph of G that contains all verticies of G.

$H = V'$
e' is a spanning subgraph of $G = (V, e)$ if $V' = V$ and $e' \leq e$

- A graph is connected if every vertex is reachable from all other verticies

has a path

- A connected component G' of Graph G is a minimal connected subgraph of G

MAXIMAL: there is no way to add into G' any verticies and/or edges of G which are not currently in G' in such a way that the resulting subgraph is connected