Running Time: (Review of lecture 19 material)

Ex. Find UB & LB of the running time of the alg. represented by

$$\sum_{i=1}^{n} \sum_{j=1}^{i^2} j^2 \cdot c = \sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2 - j)$$

**UB:**

$$\sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2 - j) \leq \sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2)$$

$$= \sum_{i=1}^{n} c(i^2)(i^2)$$

$$= \sum_{i=1}^{n} c i^4 \leq \sum_{i=1}^{n} c n^4 = c(n^4)(n)$$

- $c n^5$

**LB:**

$$\begin{align*}
\sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2 - j) & \geq \sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2 - \frac{j^2}{2}) \\
& \geq \sum_{i=1}^{n} \sum_{j=1}^{i^2} c(i^2 - \frac{j^2}{2}) \\
& = \sum_{i=1}^{n} c(i^2) \sqrt{i^2/2}
\end{align*}$$

Split the sum
The running time is

\[ C' n^5 \text{ where } \frac{C}{128} \leq C' \leq C \]
\[ f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0. \]

If \( f(n) = O(g(n)) \), the \( f(n) \) grows no faster than \( g(n) \).

\( *g(n) \) is an asymptotic upper bound (or just UB) on \( f(n) \)

\[ O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0 \} \]

The set of functions with smaller or same order of growth as \( g(n) \)

\[ f(n) \in O(g(n)) \]

\[ A \leq B \Rightarrow A + C \leq B + D \]

\[ C \leq D \]
Ex. prove that \( n^2 + n = O(n^3) \)  

sol: For \( n \geq 1 \), we have  
\[ n^2 \leq n^3 \text{ and } n \leq n^3 \]
Then \( n^2 + n \leq n^3 + n^3 = 2n^3 \)

Ex: prove that \( 3n^2 - 2n^3 + 13n - 15 = O(n^3) \)  
Pf: For \( n \geq 1 \), we have  
\[
\begin{align*}
3n^2 - 2n^3 + 13n - 15 &\leq 3n^3 + 13n \\
&\leq 3n^3 + 3n^3 \\
&\leq 6n^3
\end{align*}
\]
Ex. Prove that \( n^{3/2} + n^3 \log n = O(n^4) \)  
Pf: If \( n \geq 1 \), we have  
\[
\begin{align*}
n^{3/2} &\leq n^4 \\
\log n &\leq n \Rightarrow n^3 \log n \leq n^4 \\
n^{3/2} + n^3 \log n &\leq n^4 + n^4 = 2n^4
\end{align*}
\]