Lecture 20

Running Time (Review of Lecture 19 material)

Ex. Find UB & LB of the running time of the alg. represented by

$$\sum_{i=1}^{n}\sum_{j=1}^{i^2}\sum_{k=j}^{i^2} c = \sum_{i=1}^{n}\sum_{j=1}^{i^2} c(i^2-j)$$

UB: $$\sum_{i=1}^{n}\sum_{j=1}^{i^2} c(i^2-j) \le \sum_{i=1}^{n}\sum_{j=1}^{i^2} c(i^2)$$

$$= \sum_{i=1}^{n} c(i^2)(i^2)$$

$$= \sum_{i=1}^{n} ci^4 \le \sum_{i=1}^{n} cn^4 = c(n^4)(n)$$

$$= cn^5$$

① Split the sum

LB: $$\boxed{\sum_{i=1}^{n}\sum_{j=1}^{i^2} c(i^2-j)} \ge \sum_{i=1}^{n}\sum_{j=1}^{i^2/2} c(i^2-j)$$

$$\ge \sum_{i=1}^{n}\sum_{j=1}^{i^2/2} c(i^2-i^2/2)$$

$$= \sum^{n} c(i^2/2)(i^2/2)$$

$i=1$

$$= \sum_{i=1}^{n} c i^4/4 \geq \sum_{i=n/2}^{n} c i^4/4 \geq \sum_{i=n/2}^{n} \frac{c}{4}\left(\frac{n}{2}\right)^4$$

you get something smaller

$$= \frac{c}{64} \sum_{i=n/2}^{n} n^4 = \frac{c}{64} n^4 \left(\frac{n}{2}\right)$$

$$= \frac{cn^5}{128}$$

The running time is

$$c'n^5 \text{ where } \frac{c}{128} \leq c' \leq c$$

Asymptotic Notation

- behavior of a function as the input approaches ∞

NOTATION

- Big O - let f be a nonnegative function

$f(n)$ is O of $g(n)$, written as $f(n) = O g(n)$, iff there exist positive constants c and n_0 such that

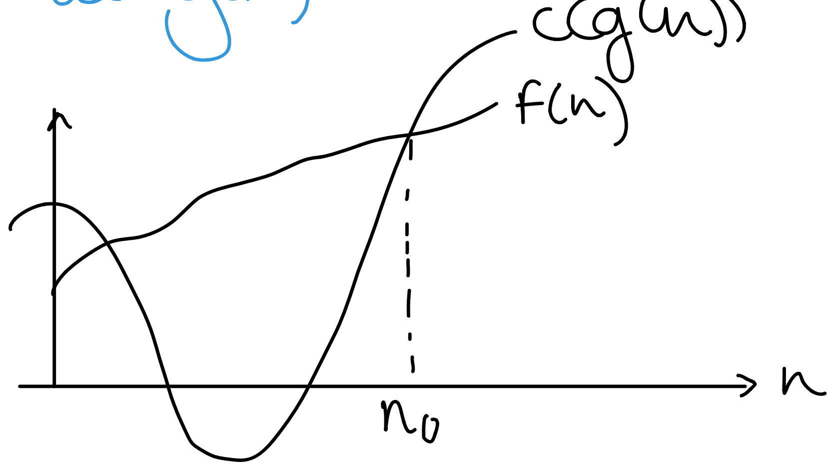
$f(n) \le c\,g(n)$ for all $n \ge n_0$.

If $f(n) = O\,g(n)$, the $f(n)$ grows no faster than $g(n)$

* $g(n)$ is an asymptotic upper bound (or just UB) on $f(n)$

$O(g(n)) = \{f(n)$: there exist positive constants c and n such that $0 \le f(n) \le c\,g(n)$ for all $n \ge n_0\}$

The set of functions with smaller or same order of growth as $g(n)$



$f(n) \in O(g(n))$

$$\left.\begin{matrix} A \le B \\ C \le D \end{matrix}\right\} \Rightarrow A + C \le B + D$$

Ex. prove that $\underbrace{n^2+n}_{f(n)} = O(\underbrace{n^3}_{g(n)})$

Sol: For $n \geq 1$, we have

no

$$n^2 \leq n^3 \text{ and } n \leq n^3$$

$$\text{Then } \underbrace{n^2+n}_{f(n)} \leq n^3 + n^3 = 2n^3$$

raise the powers

Ex: prove that $3n^3 - 2n^2 + 13n - 15 = O(n^3)$

Pf: For $n \geq 1$, we have

remove negatives

$$3n^3 - 2n^2 + 13n - 15 \leq 3n^3 + 13n$$

$$\leq 3n^3 + 13n^3 \leq 16n^3$$

Ex. Prove that $n^{7/2} + n^3 \log n = O(n^4)$

Pf: If $n \geq 1$, we have

$\log n \leq n$

$$n^{7/2} \leq n^4$$

$$\log n \leq n \Rightarrow n^3 \log n \leq n^4$$

$$n^{7/2} + n^3 \log n \leq n^4 + n^4 = 2n^4$$