Lecture 19

More examples on running time

Ex) Find the running time without line-by-line analysis

```
(1)  x = 0
(2)  for i = 1 to n do
(3)          for j = 1 to n do
(4)                  x = x + (i - j);      ← C (cost)
(5)  return(x)
```

> *Instead of doing line-by-line analysis, save time by focusing on statement #4!

If $n=3$, statement 4 runs <u>9</u> times

$n=?$, statement 4 runs n^2 times

$$\sum_{i=1}^{n}\sum_{j=1}^{n} C = \sum_{i=1}^{n} Cn(n) = Cn^2$$

Ex 2) Find running time

(1) $x = 0$

(2) for $i = 1$ to n do

(3) for $j = 1$ to i do

(4) $x = x + (i - j)$; $\longleftarrow$ Cost C

(5) return x

If $n = 3$, statement 4 runs $\underline{3}$ time

$$\sum_{k=1}^{n} \frac{n(n+1)}{2}$$

i	j
1	1
2	2
3	3
$\vdots$	$\vdots$
n	n

Arithmetic Series

$1 + 2 + 3 + \dots + n$

$$= \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)}{2}$$

$$\text{Running Time} = c\left(\frac{n(n+1)}{2}\right) \cong cn^2$$

Highest power — $\frac{n^2 + n}{2} \cong cn^2$

Ex 3) Find running time

```
(1) x = 0
(2) for i = 1 to n do
(3)     for j = i to n do
(4)         x = x + (i - j);
(5) return x;
```

Suppose $n=3$ determine # times (4) executes

i	j = i to n
1	3
2	2
3	1
$\vdots$	

i	j
1	n
2	$n-1$
3	$n-2$
$\vdots$	$\vdots$
$n-1$	2
n	1

$$(n)+(n-1)+(n-2)+\dots+2+1$$

$$= \sum_{i=1}^{n} (n-i+1) = \frac{n(n+1)}{2}$$

Running Time is $O\left(\frac{n(n+1)}{2}\right) =$

Upper/Lower Bounds for running times

- Some summations are difficult
- Bounds simplify

<u>Requires</u> - Upper bound (UB) & lower bound (LB) must be the same function only differing by a constant.

Constant for UB > constant for LB

Methods

Remove terms of expressions being subtracted, if helpful.

Substitute term (upper or lower bound of summation) into expression

LB: Split summations to reduce size.

Substitute term into expression

Example 3 (Again)

```
(1) x=0
(2) for i=1 to n do
(3)     for j=i to n do
(4)         x = x+(i-j);  → c
(5) return x;
```

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c = \sum_{i=1}^{n} c(n-i)$$

UB: $\sum_{i=1}^{n} c(n-i) \le \sum_{i=1}^{n} cn = cn(n) = cn^2$

LB: $\sum_{i=1}^{n} c(n-i) \ge ②\sum_{n/2}^{n} c(n-i) \ge \sum_{i=\frac{n}{2}}^{n} c(n-n)$

$=0$, not a good bound

Split the summation (of the first sum)

① $\sum_{i=1}^{n/2}$ or ② $\sum_{i=n/2}^{n}$

① $\ge \sum_{i=1}^{n/2} c(n-i) \ge \sum_{i=1}^{n/2} c\left(n-\frac{n}{2}\right)$

$= \sum_{i=1}^{n/2} c\left(\frac{n}{2}\right) = c\left(\frac{n}{2}\right)\left(\frac{n}{2}\right)$

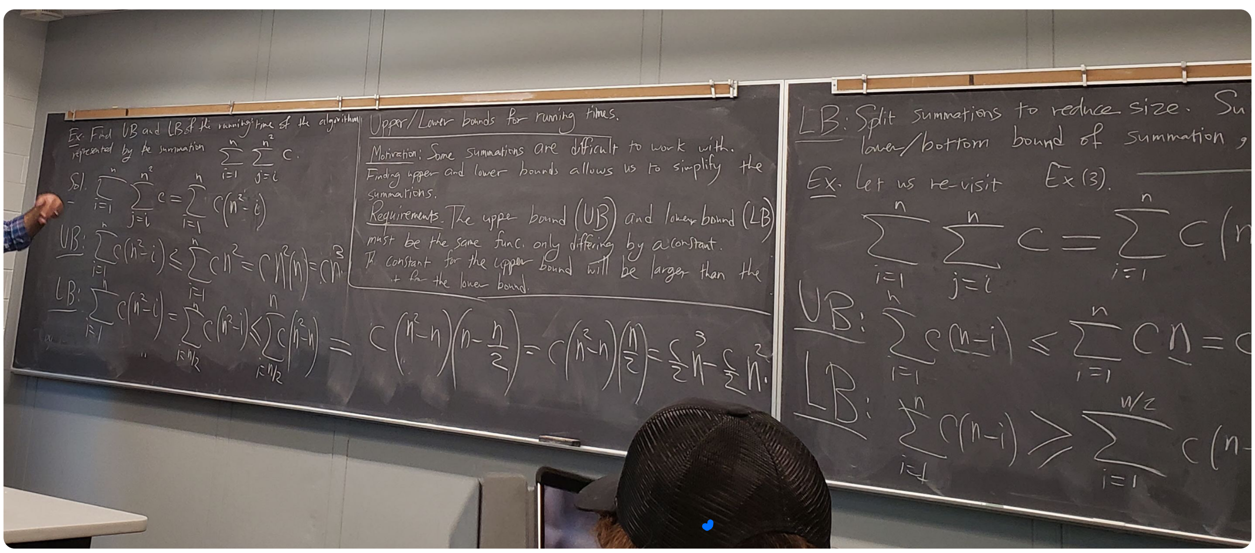
$$= \frac{c(n^2)}{4}$$

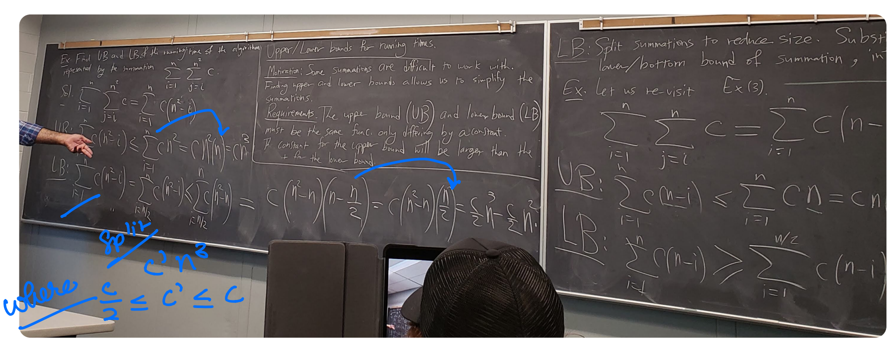
quadratic

Running Time: $c' n^2$

where $\frac{c}{4} \le c' \le c$

Example 5)

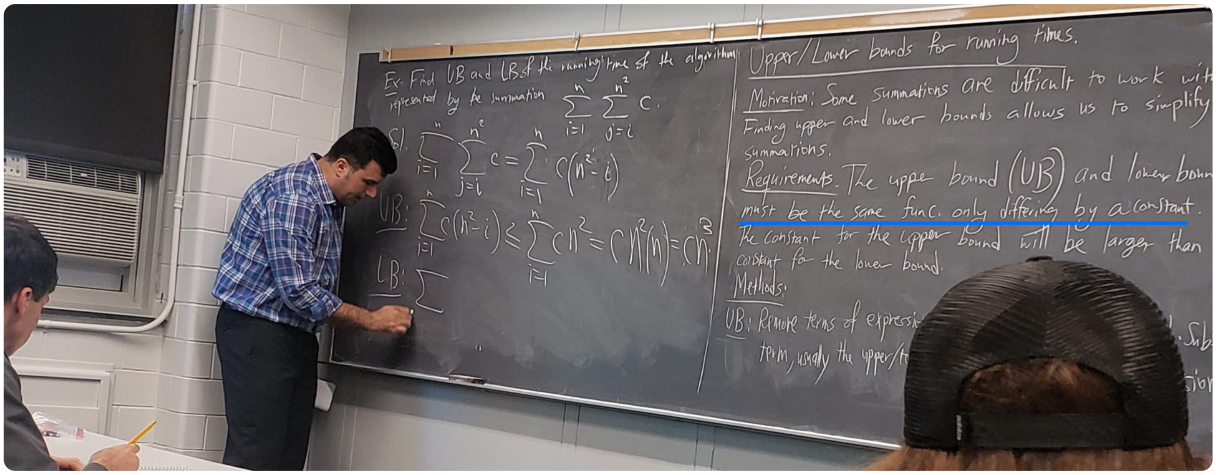




split

$c' n^3$

where $\frac{c}{2} \le c' \le c$



Homework:

https://u.osu.edu/alzalg.1/files/2019/10/hw8-1.pdf