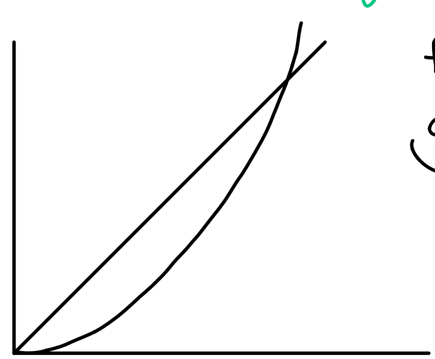
Lecture 18

Comparing Algorithms

- compare running time
  - we will see a "rough measure" that characterizes how fast the function grows (rate of growth)
- compare functions for LARGE values of n, that is, compare functions in the limit (ASYMPTOTICALLY)

Ex) choice of linear-time program and quadratic-time program



$f(n) = 100n$
$g(n) = 2n^2$

Sorting Algorithm

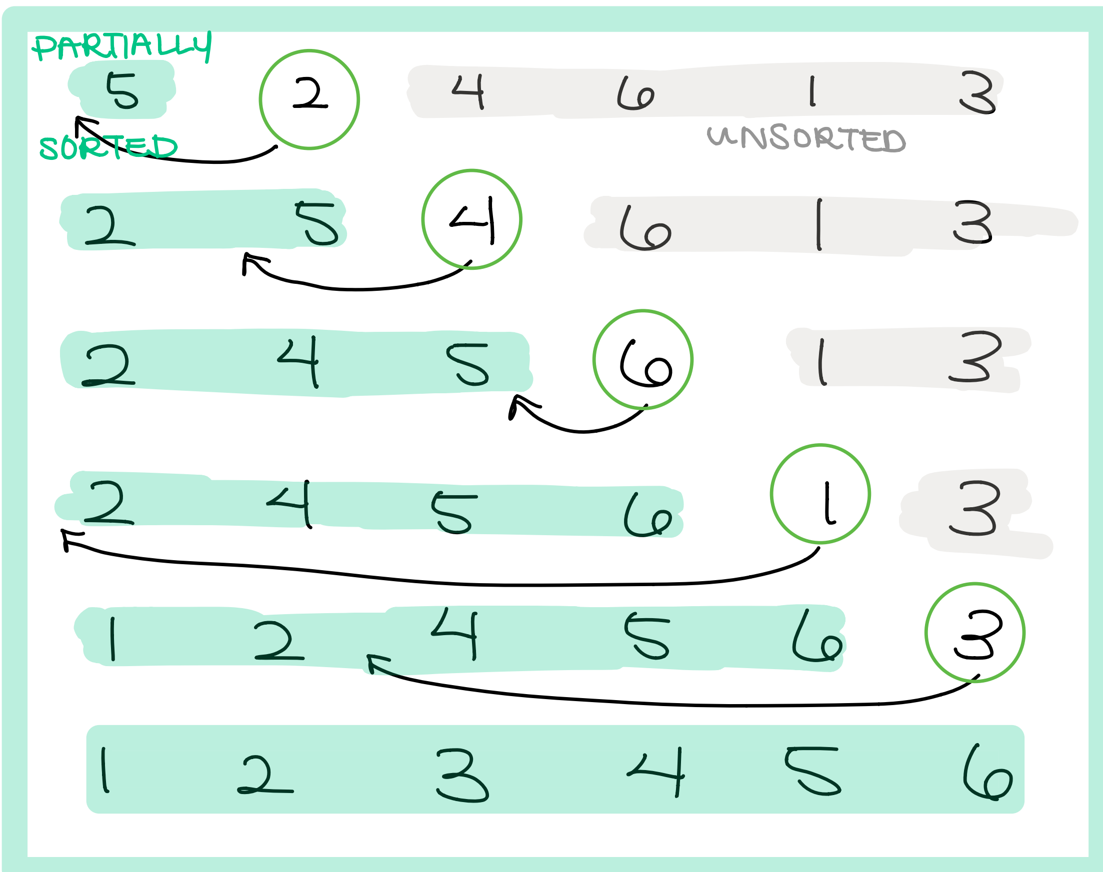
- puts elements of a list in a certain order

Ex. Insertion Sort

- how most people sort a deck of cards
- pick the next card (to be sorted),

make room for it by shifting sorted items
insert it into the correct location

Illustration : Input 5 2 4 6 1 3



Algorithm

$f(n) = \Theta n^2$

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(1)  for i=2 to n do
(2)      next = A[i]
(3)      j = i-1
(4)      while (j>0 and A[j] > next)
(5)          A[j+1] = A[j]
(6)          j = j-1;
(7)      A[i+1]= next;
```

Cost	# times		
C_1	n		
C_2	$n-1$		
C_3	$n-1$		
		BEST CASE	WORST CASE
C_4		1	$\sum_{i=2}^{n} t_i$
C_5		0	$\sum_{i=2}^{n} t_i - 1$
C_6		0	$\sum_{i=2}^{n} t_i - 1$
C_7	$n-1$		

t_i = # of times while statement is executed at iteration i

Total Cost

$$f(n) = C_1 n + C_2(n-1) + C_3(n-1) + C_4 \sum_{i=2}^{n} t_i + C_5 \sum_{i=2}^{n} t_i - 1 + C_6 \sum_{i=2}^{n} t_i - 1 + C_7(n-1)$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Best Case Analysis

- array is already sorted
  - $A[j] \leq next$ upon the first time the while loop is run. Then $t_i = 1$ for each $i = 2, \ldots, n$

$$f(n) = (C_1 + C_2 + C_3 + C_4 + C_7)n - (C_2 + C_3 + C_4 + C_7)$$

$$= k_1 n + k_2$$

$f(n) = \Theta(n)$ rate of growth is

Worst Case Analysis

- Array is in reverse order
- A[j] > next in while loop