Lecture-14

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The Scope of a variable

$P(x)$ = " x is an even # "

$Q(x)$ = " x is an odd # "

$\forall x, (P(x) \vee Q(x))$ TRUE

= "every NN is either even or odd"

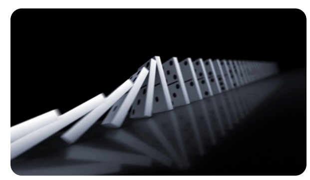
$\forall x, P(x) \vee \forall x, Q(x)$ FALSE

= "either every NN is even or every NN is odd"

can also be written as:

$\forall x, P(x) \vee \forall y, Q(y)$

$\forall x, y \ (P(x) \vee Q(y))$



Mathematical Induction

Powerful & elegant technique for proving certain types of mathematical statements

(like falling dominos)

① Prove the first statement true (knock over first domino)

② Prove if claim(k) is true, then claim(k+1) is true.

when the k^{th} domino falls it knocks over domino k+1

Infinite # of claims to prove:
claim(1), claim (2) ... claim(n)
If the steps are accomplished, we are assured all the claims are true

Principle of Mathematical Induction for Predicates

Let $P(x)$ be a sentence whose domain is $\mathbb{N}$ such that

1. $P(1)$ is true ← Base step
2. $P(k)$ is true $\Rightarrow P(k+1)$ is true, for all $k \in \mathbb{N}$

Inductive step

P is true for all $n \in \mathbb{N}$

Inductive hypothesis

Ex) prove that the sum of the first n integers is $\frac{n(n+1)}{2}$

(Show that $1+2+3+\ldots+n = \frac{n(n+1)}{2}$

1. Look for the base case

Substitute $n=1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1$$

2. State inductive hypothesis

$$1+2+3+\ldots+k = \frac{k(k+1)}{2}$$

INDUCTIVE STEP

$$P(k+1) = \underbrace{1+2+\ldots+k}_{\frac{k(k+1)}{2}} + k+1 = \frac{(k+1)(k+2)}{2}$$

$$P(k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k+1}{2}[k+2]$$