

Lecture 14

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The Scope of a variable

$P(x)$ = "x is an even #"

$Q(x)$ = "x is an odd #"

$\forall x, (P(x) \vee Q(x))$ TRUE

= "every NN is either even or odd"

$\forall x, P(x) \vee \forall x, Q(x)$ FALSE

= "either every NN is even or every NN is odd" can also be written as:

$\forall x, P(x) \vee \forall y, Q(y)$
 $\forall x, y (P(x) \vee Q(y))$



Mathematical Induction

Powerful & elegant technique for proving certain types of mathematical statements

(like falling dominoes)

- ① Prove the first statement true (knock over first domino)
- ② Prove if claim(k) is true, then claim(k+1) is true.
when the kth domino falls it knocks over domino k+1

Infinite # of claims to prove:

claim(1), claim(2) ... claim(n)

If the steps are accomplished, we are assured all the claims are true

Principle of Mathematical Induction for Predicates

Let $P(x)$ be a sentence whose domain is \mathbb{N}
such that

① $P(1)$ is true \leftarrow Base step

② $P(k)$ is true $\Rightarrow P(k+1)$ is true, for all $k \in \mathbb{N}$

P is true for all $n \in \mathbb{N}$ Inductive step

\rightarrow Inductive hypothesis

Ex) prove that the sum of the first n integers is $\frac{n(n+1)}{2}$

(show that $1+2+3+\dots+n = \frac{n(n+1)}{2}$)

① Look for the base case

Substitute $n=1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1$$

② State inductive hypothesis

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

INDUCTIVE STEP

$$P(k+1) = \underbrace{1+2+\dots+k}_{\frac{k(k+1)}{2}} + k+1 = \frac{(k+1)(k+2)}{2}$$

$$P(k+1) = \frac{k(k+1)}{2} + 2 \frac{(k+1)}{2} = \frac{k+1}{2} [k+2]$$

we show the sum of the first n integers is $\frac{n(n+1)}{2}$.
 we need to show that $1+2+3+\dots+n = \frac{n(n+1)}{2}$
 induction. Let
 $1+2+3+\dots+n = \frac{n(n+1)}{2}$
 look at Base case, see if P_0 is true.
 $1 = 1$ in both sides of equation; $1 = \frac{1(1+1)}{2} = 1$.

Next, state inductive hypothesis: Let $k \in \mathbb{N}$ such that $P(k)$ is true. That is $1+2+3+\dots+k = \frac{k(k+1)}{2}$.
 Inductive step. Using inductive hypothesis, show
 $P(k+1) = 1+2+\dots+k+(k+1) = \frac{(k+1)(k+2)}{2}$ is true.
 Now, $P(k+1) = \frac{k(k+1)}{2} + 2 \frac{(k+1)}{2} = \frac{(k+1)}{2} [k+2]$
 So, if $P(k)$ is true, then $P(k+1)$ is true for any $k > 1$.
 Conclusion For each $n \in \mathbb{N}$, $P(n)$ is true.
 In other words, for each $n \in \mathbb{N}$, $1+2+3+\dots+n = \frac{n(n+1)}{2}$.

$\sum_{k=1}^n k = \frac{n(n+1)}{2}$ $\sigma = (n+1)$

1	2	3	4	5	...
1	2	3	4	...	
1	2	3	...		
1	2	...			
1	...				

$n=5$