The Scope of a variable

\( P(x) = \text{"x is an even number"} \)
\( Q(x) = \text{"x is an odd number"} \)

\( \forall x, (P(x) \lor Q(x)) \quad \text{TRUE} \)

\( \forall x, P(x) \lor \forall x, Q(x) \quad \text{FALSE} \)

Mathematical Induction

- Powerful & elegant technique for proving certain types of mathematical statements (like falling dominos)

1. Prove the first statement true (knock over first domino)
2. Prove if claim(\( k \)) is true, then claim(\( k+1 \)) is true. When the \( k \)-th domino falls it knocks over domino \( k+1 \)

\( \text{Infinite \# of claims to prove: } \)
\( \text{claim(1)}, \text{ claim (2)} \ldots \text{ claim(n)} \)
If the steps are accomplished, we are assured all the claims are true.
Principle of Mathematical Induction for Predicates

Let \( P(x) \) be a sentence whose domain is \( \mathbb{N} \) such that

1. \( P(1) \) is true
2. \( P(k) \) is true \( \Rightarrow P(k+1) \) is true, for all \( k \in \mathbb{N} \)

\( P \) is true for all \( n \in \mathbb{N} \)

Base step

Inductive step

Inductive hypothesis

Ex) prove that the sum of the first \( n \) integers is \( \frac{n(n+1)}{2} \)

(show that \( 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2} \))

1. Look for the base case
   Substitute, \( n = 1 \)
   \[ 1 = \frac{1(1+1)}{2} \]
   \[ 1 = 1 \]

2. State inductive hypothesis
   \[ 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \]

Inductive step

\[ P(k+1) = \left( 1 + 2 + \ldots + k + (k+1) \right) = \frac{(k+1)(k+2)}{2} \]
\[ P(k+1) = \frac{k(k+1)}{2} + \frac{2(k+1)}{2} = \frac{k+1}{2} [k+2] \]