LECTURE 13

Statements in the form "All P are Q"

* "All that glitters is gold"

$GLT(x)$ = "x glitters"

$GLD(x)$ = "x is gold"

} PREDICATE

$\forall x (GLT(x) \rightarrow GLD(x))$ ✓

If x glitters, x is gold

$\forall x (\neg GLT(x) \vee GLD(x))$ ✓

$\forall x (GLT(x) \wedge GLT(x))$ ✗

Everything glitters and is gold

$\forall x \in GLT(x), GLD(x)$ ✗

Everything glitters and is gold

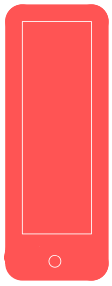
GLT is a predicate, not a domain (set)

Ex) Symbolize the negation of "All that glitters is gold"

$$\neg \forall x, (\neg GLT(x) \vee GLD(x))$$
$$\equiv \exists x, \neg(\neg GLT(x) \vee GLD(x))$$
DEMORGAN'S
$$\equiv \exists x, \neg\neg GLT(x) \wedge \neg GLD(x)$$
DOUBLE NEGATION
$$\equiv \exists x, GLT(x) \wedge \neg GLD(x)$$

Existential → universal quantifier

$$\forall x, \neg GLT(x) \vee GLD(x)$$
$$\equiv \neg\neg \forall x, \neg GLT(x) \vee GLD(x)$$
$$\equiv \neg \exists x, \neg(\neg GLT(x) \vee GLD(x))$$
$$\equiv \neg \exists x, \neg\neg GLT(x) \wedge \neg GLD(x)$$
$$\equiv \neg \exists x, GLT(x) \wedge \neg GLD(x)$$



"Nothing that glitters is gold"

$$\forall x (GLT(x) \rightarrow \neg GLD(x))$$

For all that is gold, it does not glitter

$cat(x)$ = "x is a cat"

$orange(x)$ = "x is orange"

① There are no orange cats

$$\forall x \quad cat(x) \longrightarrow \neg orange(x)$$

There is an orange cat (negation of ①)

$$\neg(\exists x, \neg cat(x) \wedge orange(x))$$

There are at least 2 cats

$$\exists x, \exists y \quad cat(x) \wedge cat(y) \quad \times$$

wrong, x & y cannot be the same cat

$$\exists x, \exists y \quad cat(x) \wedge cat(y) \wedge (x \neq y)$$

There is at most one cat (negation of there are at least 2 cats)

$$\exists x, \neg cat(x) \vee cat(x)$$

$$\neg(\exists x, \exists y \quad cat(x) \wedge cat(y) \wedge (x \neq y))$$

$$\forall x, \forall y, \neg cat(x) \vee \neg cat(y) \vee (x = y)$$

Homework:

https://u.osu.edu/alzalg.1/files/2019/09/hw5.pdf