— LECTURE 12 —

Negating Quantified Statements

FACT: The negation of a universal (existential) statement is an existential (universal) statement with the predicate negated

$$\neg \forall x, P(x) \equiv \exists x, \neg P(x)$$

$$\neg (\exists x, P(x)) \equiv \forall x, \neg P(x)$$

Ex 1 "$\underbrace{\text{Every prime \#}}_{\forall x}$ $\underbrace{\text{is odd}}_{P(x)}$" FALSE

Negation: "$\underbrace{\text{There exists a prime \#}}_{\forall x}$ $\underbrace{\text{which is not odd}}_{\neg P(x)}$"

TRUE

$\neg \forall x$

Ex 2 "There is a negative # whose square is negative."

FALSE

$P(x)$

Negation:

the square of every negative integer is nonnegative.

$\forall x$

$\neg P(x)$

Mixing Quantifiers

ALL the same quantifiers $\Rightarrow$ order doesn't matter

DIFFERENT quantifiers $\Rightarrow$ order can matter
(Universal & Existential)

① $\exists x, \exists y, P(x,y)$ ② $\exists y, \exists x, P(x,y)$

① = ②

③ $\forall x, \exists y, x+y=0$ ④ $\exists x, \forall y, x+y=0$

FALSE

2. Consider the following two propositions: (*8 points*)

(1) $$\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, P(x, y).$$ false

Some x All y

(2) $$\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, P(x, y).$$ true

All x some y

Using the above two statements, give a predicate $P(x, y)$ that makes (or explain why it is not possible):

a) Both statements true.

b) Both statements false.

c) Statement 1 true and Statement 2 false. $P(x, y) =$

d) Statement 1 false and Statement 2 true. $P(x, y) = x < y$

1
2
3
4
5
6
7

$y = 1$
$x = ?$

for all x you can find a y greater than

for every y, $y > x$
1 0

for all y

$y > x$

Negating Propositions with Multiple Quantifiers

- Negating quantified statements can be extended with more than 1 quantifier

$$\equiv \neg(\forall x \exists y \forall z, P(x,y,z))$$

$$\equiv \exists y \neg(\exists y \forall z, P(x,y,z))$$

$$\equiv \exists x \; \forall y \; \neg(\forall z, P(x,y,z))$$

$$\equiv \exists x \forall y \exists z, \neg(P(x,y,z))$$

Ex: $\mathbb{R}$ = domain

$$\forall x, \exists y, xy = 1$$

FALSE

"Every real # has a multiplicative inverse"

$$\neg(\forall x \exists y, xy = 1)$$

$$\equiv \exists x \neg(\exists y, xy = 1)$$

$$\equiv \exists x \forall y \; \neg(xy = 1)$$

$$\equiv \exists \underset{0}{x} \forall \underset{\cdot \text{ any } y}{y}, xy \neq \underset{= 0}{1}$$

Ex) $\exists x \forall y, x+y=0$

NEGATE.

$$\neg(\exists x \forall y, x+y=0)$$
$$\forall x\ \neg(\forall y, x+y=0)$$
$$\forall x \exists y, \neg(x+y=0)$$
$$\underline{\forall x \exists y, x+y \neq 0}$$

Expressing Props. using one type of quantifiers

Every universal (existential) statement can be expressed as an existential (universal) statement

Changing $\forall$ to $\exists$: $\forall x, P(x) \equiv \neg\neg\forall x\ P(x) \equiv \neg\exists x, \neg P(x)$