Lecture 10

Universal sets of Logical Operators

A set of operators, such as $\{\wedge, \vee, \neg\}$, that can be used to express any proposition is called universal

NOTE: The set of operators in $\{\wedge, \neg\}$ is also universal, as $P \vee Q$ can be expressed using just $\neg$ and $\wedge$

$= P \vee Q \overset{DNL}{\equiv} \neg\neg(P \vee Q) \equiv \neg(\neg P \wedge \neg Q)$

Ex) $P \vee (\neg Q \wedge R) \equiv \neg(\neg P \wedge \neg(\neg Q \wedge R))$

Exc ① Prove that the set of operators $\{\vee, \neg\}$ is universal

② " " is NOT universal

Predicate Logic

A predicate is a proposition whose truth value depends on the value of one or more variables

Ex) ① $P(x) = $ "$x > 4$"

Predicate on one variable

② $Q(x,y) = "x \geq y"$

As $Q(5,4)$ is true, but $Q(4,5)$ is false

Quantifiers

An expression (such as "every", "there exists") that indicates the scope of a term to which it is attached

① Every prime # greater than 2 is odd.
quantifier

② There exists an even prime #
quantifier

③ For all integers greater than 0, there are no positive integers a, b, and c that satisfy $a^n + b^n = c^n$
quantifiers

"ALL"

$\forall$ = "every" or "for all" and are called UNIVERSAL QUALIFIERS

$\exists$ = "there is" / "there exists" and are called EXISTENTIAL QUALIFIERS

"Exists"

Ex) Let $P(x) =$ " $x^2 > 4$ ", then

$\mapsto 1, 2, 3, 4, 5, 6$

① $\exists x \in \mathbb{N}, P(x)$ is a prop that means

"there is a natural number x such that $x^2 > 4$ " TRUE

② $\forall x \in \mathbb{N}, P(x)$ is a prop that means

"for every natural number x we have $x^2 > 4$ " FALSE

Universe of Discourse / Domain of Discourse / D

The set of the possible values of the variables in the predicate

IMPORTANT TO STATE THE DOMAIN

The simplest form of quantified formula is

A quantifier, a variable, a predicate

① We write $\forall x \in D, P(x)$

= "A Predicate $P(x)$ is true for all values of x in some set D

For all $x \in D$, $P(x)$ is true

② $\exists x \in D, P(x)$

= "A predicate $P(x)$ is true for at least one value of x in D

There exists $x \in D$ such that $P(x)$ is true

Ex : $\exists\, n \in \mathbb{N},\ n = n^2$

Some natural # is equal to its own square

Homework:

https://u.osu.edu/alzalg.1/files/2019/09/hw4.pdf