Lecture 11

Multiple Predicate

Some propositions have more than one predicate.

Ex) "Some NN is equal to its own square and is equal to its own cube"

$$\exists\, n \in \mathbb{N},$$
$$n = n^2 \wedge n = n^3$$

Multiple Quantifiers

More than one quantifier can be used in a prop.

Ex) Let $P(x,y)$ = "$x + y = 5$"

① There is an NN x and an NN y such that $x + y = 5$

OR There exists two NNs whose sum is 5

$$\exists\, x \in \mathbb{N},$$
$$\exists\, y \in \mathbb{N},\quad P(x,y) \qquad \underline{\text{TRUE}}$$

② $\forall\, x \in \mathbb{N},\ \forall\, y \in \mathbb{N}$

For every NN x & every NN y,
$x + y = 5$ $\qquad$ FALSE

Ex) Let $Q(x,y) =$ "$x+y=2$" Then

① The prop $\exists x \in \mathbb{N}, \exists y \in \mathbb{N}, Q(x,y)$

TRUE

② The prop $\exists x \in \mathbb{N}, \exists y \in \mathbb{N},$
$Q(x,y) \wedge x \neq y$

FALSE

0 is not a NN

IMPORTANT:
② Becomes true when you change the domain from $\mathbb{N}$ to $\mathbb{R}$

Other Examples

① $\forall x \in \mathbb{N}, \forall y \in \mathbb{N}, \quad xy \geq 0$

TRUE

② $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy \geq 0$

FALSE, $x = -2$
$y = 3$ $\quad xy = -6$

Goldbach's Conjecture

"Every even integer n greater than 2 is the sum of two primes."

↳ For every even int $n > 2$, there exists primes p & q such that $n = p + q$

EVENS = set of even ints >2

PRIMES = set of prime #s

$\forall x \in$ Evens, $\exists p \in$ Primes, $\exists q \in$ Primes, $\underline{n = p + q}$

↳ or $\exists p, q \in$ Primes

$P(x)$ & $Q(x)$ are 2 predicates such that ① & ② and ③-④ are logically $\equiv$

① $\exists x \in D, P(x) \wedge Q(x)$

② $\exists x, P(x) \wedge Q(x)$

③ $\exists x \in D, P(x) \wedge \exists x \in D, Q(x)$

④ $\exists x, P(x) \wedge \exists x, Q(x)$

⑤ $\exists x \in D, P(x) \wedge \exists y \in D, Q(y)$

⑥ $\exists x, y \in D, P(x) \wedge Q(y)$

⑦ $\exists x, y, P(x) \wedge Q(y)$

Negating quantified statements