CSE 2321 Practice Exercises - First Exam

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Question 1.

[4 points] Which of the following sentences are propositions? What are the truth values of those that are propositions? (Here, in items 3 and 5, x and y are any real numbers).

1. $2 + 3 = 5$.
2. $5 + 7 = 10$.
3. $x + 2 = 11$.
4. Your answer to item 3 is incorrect.
5. $x + y = y + x$.
6. Do not submit the solution of this homework.
7. What time is it?
8. John F. Kennedy is the 35th president of the United States.

Question 2.

[3 points] What is the negation of each of the following propositions?

1. Today is Thursday.
2. $2 + 1 = 3$.
3. The summer in Santiago is not hot.

Question 3.

[4 points] Let P and Q be propositions

P : I bought a lottery ticket this week.
Q : I won the million dollar jackpot on Friday.

Express each of the following propositions as an English sentence.

1. $\neg P$.
2. $P \vee Q$.
3. $P \wedge Q$.
4. $\neg P \wedge \neg Q$.

Question 4.

[4 points] Let P and Q be propositions

P : It is below freezing.
Q : It is snowing.

Write the following propositions using P and Q and logical connectives.

1. It is below freezing and snowing.
2. It is below freezing but not snowing.
3. It is not below freezing and it is not snowing.
4. It is either snowing or below freezing (or both).

Question 5.

[11 points] Construct truth tables for the compound propositions:

1. $P \wedge \neg P$.
2. $(P \vee \neg Q) \to Q$.
3. $(P \vee Q) \to (P \wedge Q)$.
4. $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$.
5. $P \oplus P$.
6. $P \oplus \neg Q$.
7. $\neg P \oplus \neg Q$.
8. $(P \oplus Q) \wedge (P \oplus \neg Q)$.
9. $P \to \neg Q$.
10. $\neg P \leftrightarrow Q$.
11. $(P \leftrightarrow Q) \vee (\neg P \leftrightarrow Q)$.
12. $(P \wedge Q) \vee R$.
13. $(P \wedge Q) \vee \neg R$.
14. $P \to (\neg Q \vee R)$.
15. $(P \to Q) \wedge (\neg P \to R)$.
16. $(P \leftrightarrow Q) \vee (\neg Q \leftrightarrow R)$.

Question 6.

[3 points] Write the contrapositive, converse, and inverse of the proposition: "If the Sun is shrunk to the size of your head, then the Earth will be the size of the pupil of your eye".

Question 7.

[1 point] What is the negation of the following proposition?

"If the Sun is shining in Santiago's sky, then I will go to the nearest beach and do a little physical exercise."

Question 8.

[4+4 points] Determine if each of the following implications is a tautology or a contradiction. Justify your answer without using truth tables.

1. $[\neg P \wedge (P \vee Q)] \rightarrow Q$.
2. $[P \wedge (P \rightarrow Q)] \rightarrow Q$.

Question 9.

[4 points] Construct a *disjunctive normal form* having the following truth table.

P	Q	R	Statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

Question 10.

[3 point] For each of the following statements, write an equivalent statement in conjunctive normal form.

1. $\neg(A \vee B)$.
2. $\neg(A \wedge B)$.
3. $A \vee (B \wedge C)$.

Question 11.

[5 points] Let the domain of discourse consist of the non-zero integers, i.e., $D = \mathbb{Z} - \{0\}$. Give the truth value of each of the following quantified statements. Find a value of x which supports your conclusion.

1. $\forall x \in D, x < 2x$.
2. $\exists x \in D, x + x = x - x$.
3. $\exists x \in D, x^2 = 2x$.
4. $\forall x \in D, \frac{x}{2x} < x$.
5. $\exists x \in D, \frac{x}{x} = x$.

Question 12.

[6 points] Consider the following two propositions.

$$\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, P(x, y), \tag{1}$$

$$\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, P(x, y). \tag{2}$$

Using the above two propositions, give a predicate $P(x, y)$ that makes (or explain why it is not possible):

1. Propositions (1) and (2) are both true.

2. Propositions (1) and (2) are both false.
3. Proposition (1) is true and Proposition (2) is false.
4. Proposition (1) is false and Proposition (2) is true.

Question 13.

[4 points] Negate the false proposition $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x + y = 0$.

Question 14.

[2 points] Find the number of subsets of the given set. Justify your answer.

$$T = \{n \in \mathbb{N} : n \text{ is an even number between 21 and 41}\}.$$

Question 15.

[8 points] Use mathematical induction to prove that the cardinality of the powerset of a finite set A is equal to 2^n if the cardinality of A is n. Show all the steps in the proof.