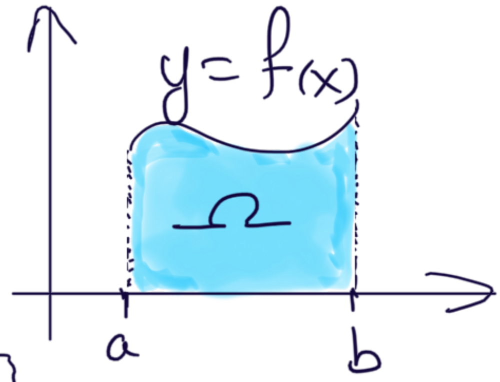
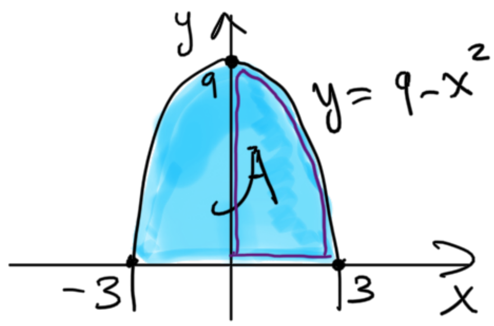
Area

The area of the region bounded by the graph of $f(x)$ and the x-axis between the lines $x=a$ and $x=b$



$$\boxed{\text{Area of } \Omega = \int_a^b f(x)\,dx.}$$

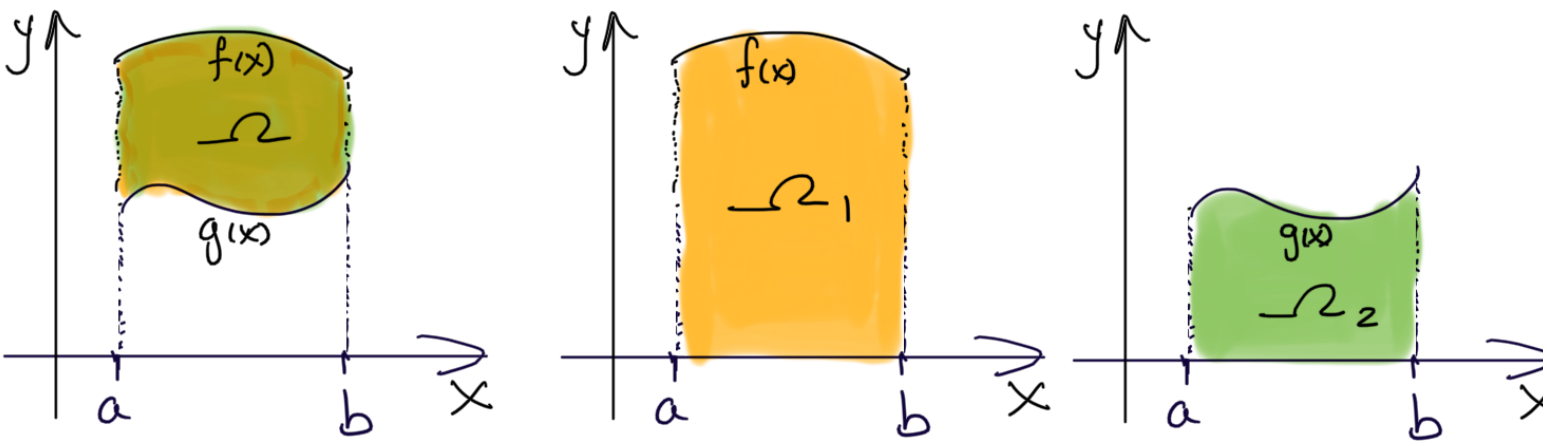
Ex. Find the area of the region bounded above by the curve $y = 9 - x^2$ and below by the x-axis.



Soln.
$$A = \int_{-3}^{3} (9 - x^2)\,dx$$
$$= 9x - \frac{x^3}{3} \Big|_{-3}^{3}$$
$$= (27 - 9) - (-27 + 9)$$
$$= 36.$$

Or $A = 2\int_0^3 (9 - x^2)\,dx = 2(18) = 36.$

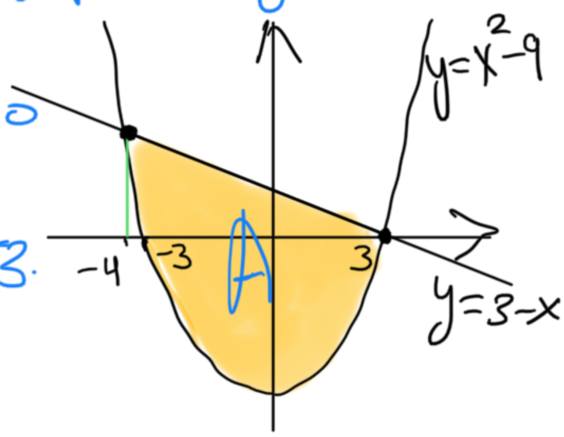
Area between curves.



$$\Omega = \Omega_1 - \Omega_2 = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$\boxed{\Omega = \int_a^b \left(f(x) - g(x)\right) dx}$$

Ex. Find the area bounded by the graphs of $y = 3 - x$ and $y = x^2 - 9$.



Soln. If $3 - x = x^2 - 9$, then $x^2 + x - 12 = 0$ and hence $(x-3)(x+4) = 0$.

So, the curves intersect when $x = -4$ & 3.

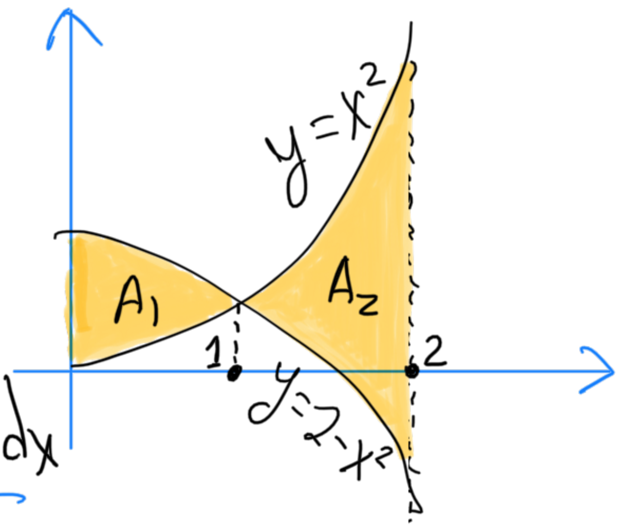
Now, $A = \int_{-4}^{3} \left[(3-x) - (x^2-9)\right] dx$

$= \int_{-4}^{3} \left(-x^2 - x + 12\right) dx$

$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 12x\right]_{-4}^{3}$

$= 343/6$.

Ex. Find the area bounded by the graphs of $y = x^2$ and $y = 2 - x^2$ for $0 \le x \le 2$.



Soln. If $x^2 = 2 - x^2$, then $\cancel{2}x^2 = \cancel{2}\,1$

So $x = \pm 1$.

$$A = \int_0^1 \underbrace{\left[(2-x^2) - x^2\right]}_{A_1} dx + \int_1^2 \underbrace{\left[x^2 - (2-x^2)\right]}_{A_2} dx$$

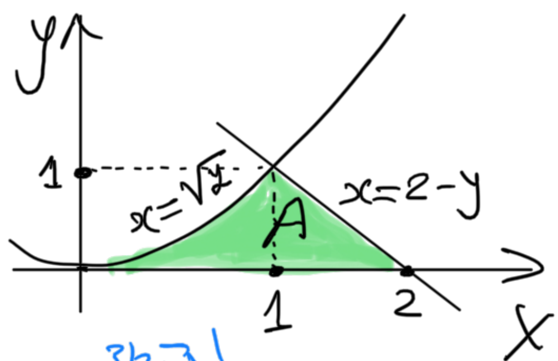
$$= \int_0^1 (2 - 2x^2)\,dx + \int_1^2 (2x^2 - 2)\,dx$$

$$= \left[2x - \frac{2x^3}{3}\right]_0^1 + \left[\frac{2x^3}{3} - 2x\right]_1^2$$

$$= \left(2 - \frac{2}{3}\right) - (0 - 0) + \left(\frac{16}{3} - 4\right) - \left(\frac{2}{3} - 2\right)$$

$$= 4.$$

Ex. Find the area bounded by the graphs of $y = x^2$, $y = 2 - x$ and $y = 0$.

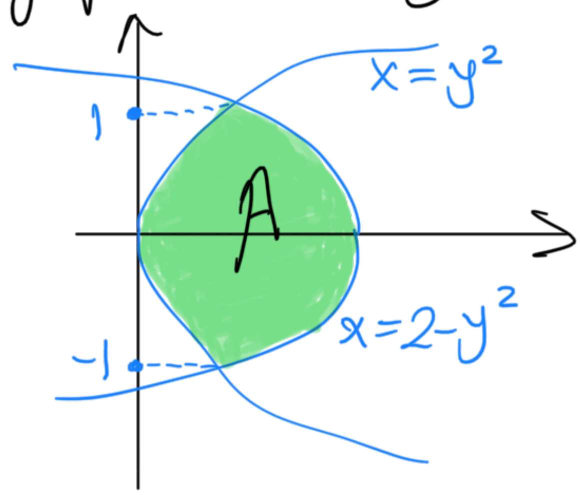


Soln. $y = 2 - x$, or $x = 2 - y$.

$y = x^2$, or $x = \pm\sqrt{y}$.

$$A = \int_0^1 \left[(2-y) - \sqrt{y}\right] dy = \left[2y - \frac{1}{2}y^2 - \frac{2}{3}y^{3/2}\right]_0^1 = 5/6.$$

Ex. Find the area bounded by the graphs of $x=y^2$ and $x=2-y^2$.



Soln. If $y^2=2-y^2$ then $\cancel{2}y^2=\cancel{2}^{\,1}$.

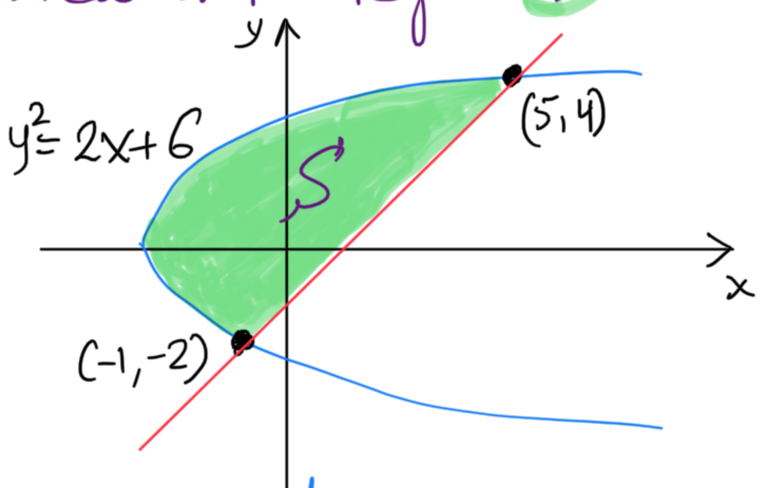
$$A=\int_{-1}^{1}\left[(2-y^2)-y^2\right]dy$$

$$=\int_{-1}^{1}(2-2y^2)\,dy$$

$$=\left[2y-\frac{2}{3}y^3\right]_{-1}^{1}$$

$$=8/3 .$$

Ex. Find the area of the region S show below.



Soln. The equation of the parabola can be written as

$$\boxed{x=\frac{1}{2}y^2-3.}$$

The equation of the line segment that passes through the points $(-1,-2)$ and $(5,4)$ is

$$y=\frac{4-(-2)}{5-(-1)}x+b=x+b$$

$\boxed{(x,y)=(5,4)}$ $\Longrightarrow$ $5=4+b$. So $b=1$. Thus $\boxed{x=y+1}$

The area of S is given by

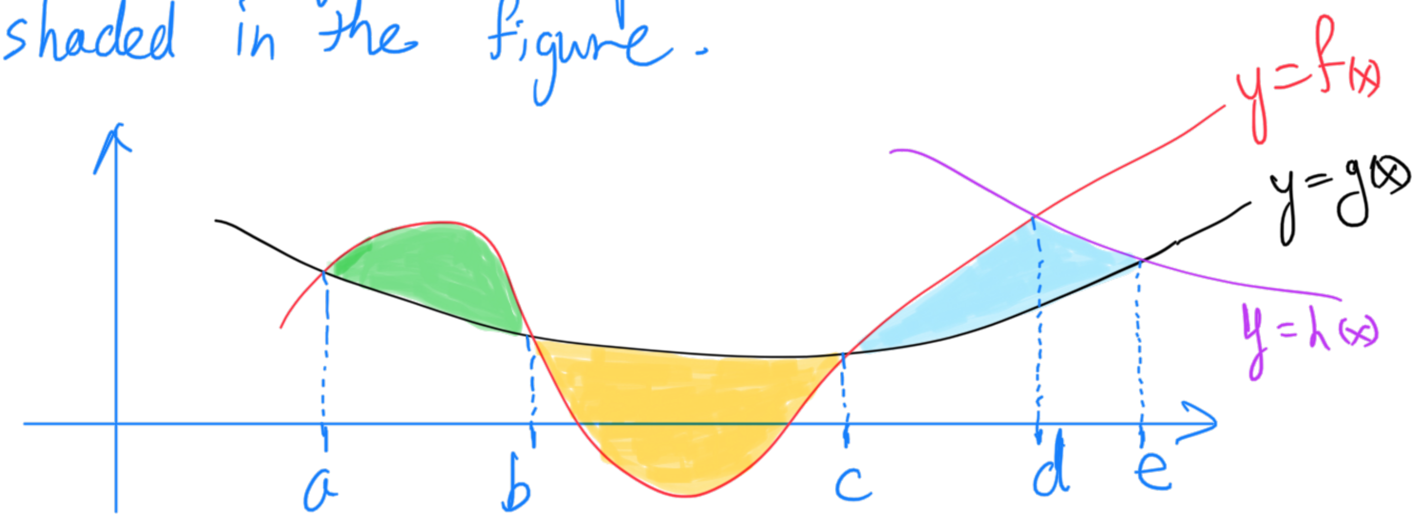
$$A = \int_{-2}^{4} \left[(y+1) - \left(\tfrac{1}{2}y^2 - 3\right)\right] dy$$

$$= \int_{-2}^{4} \left(-\tfrac{1}{2}y^2 + y + 4\right) dy$$

$$= -\frac{1}{2}\frac{y^3}{3} + \frac{y^2}{2} + 4y \Big|_{-2}^{4}$$

$$= 18.$$

Ex. Use integrals to represent the area of the region shaded in the figure.



Soln. $A = \int_a^b \left(f(x) - g(x)\right) dx + \int_b^c \left(g(x) - f(x)\right) dx$

$+ \int_c^d \left(f(x) - g(x)\right) dx + \int_d^e \left(h(x) - g(x)\right) dx.$

This lecture: Areas.

Next lecture: Volumes.

Searching keywords:

- Find the area bounded احسب المساحة المحصورة
- The University of Jordan الجامعة الأردنية
- Calculus II 2 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/102.aspx

For any comments or concerns, please use my email to contact me.

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