Taylor and Maclaurin series.

Which functions have power series representation?
And how can we find such representation?
In this section, we find power series representations for a certain class of functions.

Taylor's theorem.

Suppose that f has $(n+1)$ continuous derivatives on an open interval I containing 0. Then for each $x \in I$, $f(x) = P_n(x) + R_{n+1}(x)$, where

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!} x^k = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

and $R_{n+1}(x) = \dfrac{f^{(n+1)}(c)}{(n+1)!} x^n$, where c is some number between 0 and x.

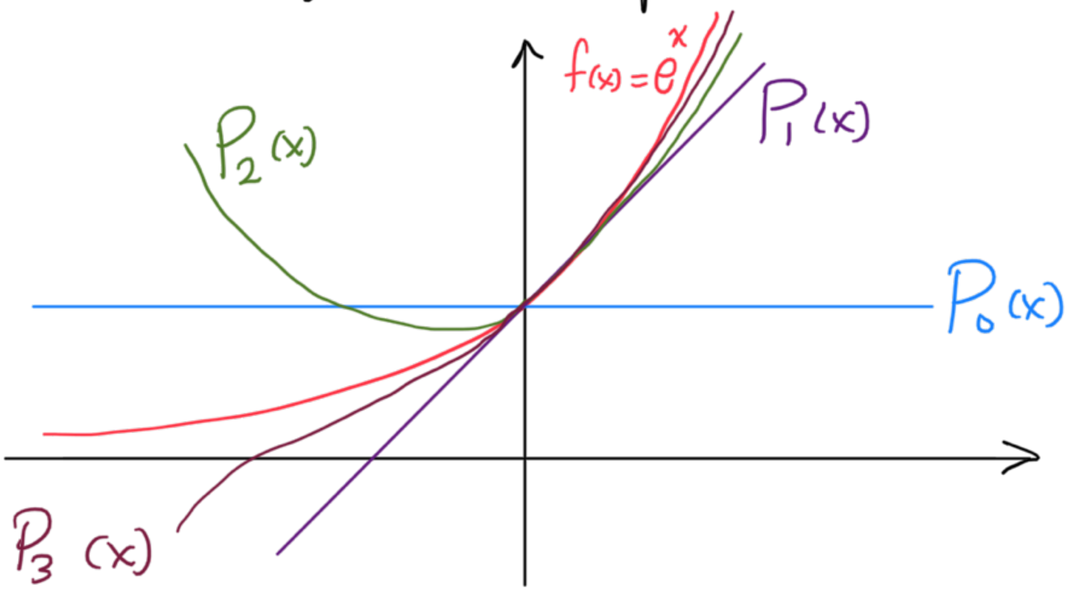
Def. In Taylor's thm, the func. $P_n(x)$ is called the n^{th}-degree Taylor polynomial of f at 0, and the term $R_{n+1}(x)$ is the remainder term.

Remark: If $R_{n+1}(x) \xrightarrow{\text{as } n\to\infty} 0$ for each $x \in I$, then $P_n(x) \longrightarrow f(x)$ for each $x \in I$.

In this case,

$$\boxed{f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \cdots}$$

This series is called the Taylor series of f at 0. It is also given the special name Maclaurin series.



Note that as n increases $P_n(x)$ appears to approach the func. e^x.

Ex. For each of the following functions, $R_{n+1}(x) \longrightarrow 0$ as $n \longrightarrow \infty$. Find the Taylor series of f at 0 for f (or the Maclaurin series for f).

(1) $f(x) = e^x$.

Soln. $f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$

k	0	1	2	3	4	$\cdots$
$f^{(k)}(x)$	e^x	e^x	e^x	e^x	e^x	$\cdots$
$f^{(k)}(0)$	1	1	1	1	1	$\cdots$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \cdots$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots$$

Thus, $\boxed{e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for all } x.}$

In particular, $e = \sum_{k=0}^{\infty} \frac{1}{k!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$

(2) $f(x) = \sin x$.

Soln.

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$= \cancel{f(0)}^{0} + f'(0)x + \frac{\cancel{f''(0)}^{0}}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$+ \frac{\cancel{f^{(4)}(0)}^{0}}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \frac{\cancel{f^{(6)}(0)}^{0}}{6!}x^6 + \frac{f^{(7)}(0)}{7!}x^7$$

$$+ \cdots.$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

k	$f^{(k)}(x)$	$f^{(k)}(0)$
0	$\sin x$	0
1	$\cos x$	1
2	$-\sin x$	0
3	$-\cos x$	-1
4	$\sin x$	0
5	$\cos x$	1
6	$-\sin x$	0
7	$-\cos x$	-1
$\vdots$	$\vdots$	$\vdots$

Thus, $\boxed{\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}, \text{ for all } x}$.

(3) $$\boxed{\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}, \text{ for all } x}$$ Exc.

(4) $$\boxed{\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} x^k, \; x \in (-1,1)}$$ Exc.

(5) $$\boxed{\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} x^{2k+1}, \; x \in (-1,1)}$$ Exc.

<u>Ex</u>. Find the sum.

(1) $$\sum_{k=0}^{\infty} \frac{5^k}{k!} = e^5.$$

---

(2) $$\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{3}\right)^{2k+5}}{(2k+1)!} = \left(\frac{\pi}{3}\right)^4 \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{3}\right)^{2k+1}}{(2k+1)!}$$

$$= \left(\frac{\pi}{3}\right)^4 \sin\left(\frac{\pi}{3}\right)$$

$$= \left(\frac{\pi}{3}\right)^4 \left(\frac{\sqrt{3}}{2}\right).$$

---

(3) $$\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k-3}}{(2k)!} = \left(\frac{\pi}{2}\right)^{-3} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k}}{(2k)!}$$

$$= \left(\frac{\pi}{2}\right)^{-3} \cos\left(\frac{\pi}{2}\right)$$

$$= \text{zero}.$$

(4) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(k+1)\,3^{k+1}} = \sum_{n=1}^{\infty} \frac{(-1)^{n+2}}{n\,3^n}$ $(n=k+1)$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\left(\frac{1}{3}\right)^n$$

$$= \ln\left(1+\frac{1}{3}\right) = \ln\left(\frac{4}{3}\right).$$

---

(5) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)\,2^{4k}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\,\frac{4}{2^{4k+2}}$

$$= 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\,\frac{1}{4^{2k+1}}$$

$$= 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}\left(\frac{1}{4}\right)^{2k+1}$$

$$= 4\tan^{-1}\left(\frac{1}{4}\right).$$

$$\cong 4\pi/13.$$

---

(6) $\dfrac{1}{1.2} - \dfrac{1}{2.2^2} + \dfrac{1}{3.2^3} - \dfrac{1}{4.2^4} + \cdots\cdots$ Exc.

Final Ans. $\ln(3/2)$.

Ex. Find the Maclaurin series for the func.

(1) $f(x) = e^{x^2+1}$.

$$\underline{\text{Soln.}}\quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\xrightarrow{\text{Replace } x \text{ by } x^2} \quad e^{x^2} = \sum_{k=0}^{\infty} \frac{(x^2)^k}{k!} = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}.$$

$$\text{Then } e^{x^2+1} = e\,e^{x^2} = e\sum_{k=0}^{\infty} \frac{x^{2k}}{k!}, \quad x \in \mathbb{R}.$$

---

(2) $f(x) = x^2 \cos(x^3)$.

$$\underline{\text{Soln.}}\quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

$$\xrightarrow{\text{Replace } x \text{ by } x^3} \quad \cos x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (x^3)^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{6k}.$$

$$\text{Then}\quad x^2 \cos x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{6k+2}, \quad x \in \mathbb{R}.$$

---

(3) $f(x) = \sin\left(x^2 - \frac{\pi}{2}\right)$.

Recall that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\underline{\text{Soln.}}\quad f(x) = \sin x^2 \underbrace{\cancel{\cos\frac{\pi}{2}}}_{0} - \sin\frac{\pi}{2}\cos x^2$$

$$= -\cos x^2$$

$$= (-1)\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (x^2)^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{4k}, \quad x \in \mathbb{R}.$$

(4) $f(x) = \cosh x$.

Soln. $\cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right)$.

$$e^{x} = 1 + \cancel{x} + \frac{x^2}{2!} + \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} + \cancel{\frac{x^5}{5!}} + \cdots$$

$$+ \quad e^{-x} = 1 - \cancel{x} + \frac{x^2}{2!} - \cancel{\frac{x^3}{3!}} + \frac{x^4}{4!} - \cancel{\frac{x^5}{5!}} + \cdots$$

$$e^{x} + e^{-x} = 2 + 2 \cdot \frac{x^2}{2!} + 2 \cdot \frac{x^4}{4!} + \cdots$$

Then $\cosh x = \frac{1}{2}\left[e^{x} + e^{-x}\right]$

$$= \frac{1}{\cancel{2}}\left[\cancel{2} + \cancel{2} \cdot \frac{x^2}{2!} + \cancel{2} \cdot \frac{x^4}{4!} + \cdots\right]$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

Then $\boxed{\cosh x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}, \quad x \in \mathbb{R}}$.

(5) $\boxed{\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}, \quad x \in \mathbb{R}.}$ Exc.

The binomial series.

Recall that $\binom{k}{n} = \frac{k!}{(k-n)!\, n!}$

For example $\binom{4}{2} = \frac{4!}{(4-2)!\,2!} = \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot \cancel{2}} = 6$.

Binomial series: If $\alpha \in \mathbb{R}$ and $|x| < 1$, then

$$\boxed{(1+x)^{\alpha} = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \cdots}$$

Ex. Find the Maclaurin series for the func. $f(x) = \frac{1}{\sqrt{9-x}}$ and its radius of convergence (R.C.).

Soln. $$\frac{1}{\sqrt{9-x}} = \frac{1}{\sqrt{9\left(1-\frac{x}{9}\right)}} = \frac{1}{3\sqrt{1-\frac{x}{9}}} = \frac{1}{3}\left(1-\frac{x}{9}\right)^{\overset{\alpha}{-1/2}}.$$

$$= \frac{1}{3} \sum_{k=0}^{\infty} \binom{-\frac{1}{2}}{k} \left(\frac{-x}{9}\right)^k.$$

This series converges for $\left|-\frac{x}{9}\right| < 1$, that is for $|x| < 9$. Thus, the R.C. is $r = 9$.

Ex. Evaluate the integral as an infinite series.

(1) $\int e^{x^2} dx$.

Soln. $$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} \xrightarrow{\text{Replace } x \text{ by } x^2} e^{x^2} = \sum_{k=0}^{\infty} \frac{x^{2k}}{k!}.$$

Then $$\int e^{x^2} dx = \sum_{k=0}^{\infty} \frac{1}{k!} \int x^{2k}\, dx$$

$$= \sum_{k=0}^{\infty} \frac{1}{k!(2k+1)} x^{2k+1} + C.$$

---

(2) $\int e^{-x^2} dx$. Exc.

(3) $\int \ln x \, e^x \, dx$.

Soln. $\int \ln x \, e^x \, dx = \int \ln x \left(\sum_{k=0}^{\infty} \frac{x^k}{k!} \right) dx$

$$= \sum_{k=0}^{\infty} \frac{1}{k!} \int x^k \ln x \, dx$$

For $\int x^k \ln x \, dx$, we let

dv		u	
x^k	$\longrightarrow$	$\ln x$	$+$
$\frac{x^{k+1}}{k+1}$	$\longrightarrow$	$\frac{dx}{x}$	$-$

$$= (\ln x)\left(\frac{x^{k+1}}{k+1}\right) - \frac{1}{k+1}\int x^k \, dx$$

$$= \ln x \, \frac{x^{k+1}}{k+1} - \frac{x^{k+1}}{(k+1)^2} + C_1.$$

Then $\int \ln x \, e^x \, dx = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\ln x \, \frac{x^{k+1}}{k+1} - \frac{x^{k+1}}{(k+1)^2} + C_1 \right]$

$$= \ln x \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)k!} - \sum_{k=0}^{\infty} \frac{x^{k+1}}{k!\,(k+1)^2} + C_2,$$

where $C_2 = C_1 \sum_{k=0}^{\infty} \frac{1}{k!} = C_1 e$.

Ex. Evaluate

(1) $\lim_{x \to 0} \frac{\sin x^3 - x^3}{x^9}$.

Soln. $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$

$\boxed{x^3 \longrightarrow x}$ $\Rightarrow$ $\sin x^3 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (x^3)^{2k+1}$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{6k+3}$$

$$= x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \cdots$$

Then $\frac{\sin x^3 - x^3}{x^9} = \frac{\left(\cancel{x^3} - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \cdots\right) - \cancel{x^3}}{x^9}$

$$= \frac{-1}{3!} + \frac{x^6}{5!} + \cdots$$

Thus, $\lim_{x \to 0} \frac{\sin x^3 - x^3}{x^9} = \frac{-1}{3!} = -\frac{1}{6}$.

(2) $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$. Exc. Find Ans. is $\frac{1}{2}$.

Ex. Use series to show that $\frac{d}{dx} \cos x = -\sin x$.

Proof: $\frac{d}{dx} \cos x = \frac{d}{dx}\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\right]$

$$= \frac{-\cancel{2}x}{\cancel{2!}} + \frac{\cancel{4}x}{\underset{3!}{\cancel{4!}}} - \frac{\cancel{6}x}{\underset{5!}{\cancel{6!}}} + \frac{\cancel{8}x^8}{\underset{7!}{\cancel{8!}}} - \cdots$$

$$= -\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right]$$

$$= -\sin x.$$

Ex. Let $f(x) = x \sin x^2$. Find (A) $P_4(x)$. (B) $f^{(19)}(0)$.

Soln. $\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$.

$\boxed{x^2 \to x} \Rightarrow \sin x^2 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} (x^2)^{2k+1}$. Then

$$x \sin x^2 = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+3}$$

$$= \underset{k=0}{\boxed{}} x^3 + \underset{k=1}{\boxed{}} x^7 + \underset{k=2}{\boxed{}} x^{11} + \underset{k=3}{\boxed{}} x^{15} + \underset{k=4}{\boxed{*}} x^{19} + \underset{k=5}{\boxed{}} x^{23} + \cdots$$

(A) $P_4(x) = x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \frac{x^{15}}{7!} + \frac{x^{19}}{9!}$.

(B) Note that $\frac{d^{19}}{dx^{19}}\left[\frac{(-1)^k x^{4k+3}}{(2k+1)!}\right] = \begin{cases} \text{Zero}, & \text{for } k=4, \\ \frac{(-1)^4 (4(4)+3)!}{(2(4)+1)!}, & \text{for } k=4. \end{cases}$

This is $19!/9!$

It follows that $f^{(19)}(0) = 19!/9!$.

Taylor series centered at a.

Taylor's thm (centered at a).

Suppose that g has $(n+1)$ continuous derivatives on an open interval I containing the point a.

Then for each $x \in I$, we have

$$g(x) = \sum_{k=0}^{n} \frac{g^{(k)}(a)}{k!}(x-a) + R_{n+1}(x), \text{ where}$$

$$R_{n+1}(x) = \frac{g^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} \left(\begin{array}{l} c \text{ is some number between} \\ x \text{ and } a. \end{array}\right).$$

Remark: If $R_{n+1}(x) \longrightarrow 0$ as $n \longrightarrow \infty$

then $\boxed{g(x) = \sum_{k=0}^{\infty} \frac{g^{(k)}(a)}{k!}(x-a)^k}$.

This series is called the Taylor series of the func. f at a.

Ex. Find the Taylor series for the func. at a.

(1) $g(x) = 4x^3 - 3x^2 + 5x - 1$, at $a = 2$.

Soln. $g(x) = 4x^3 - 3x^2 + 5x - 1 \Longrightarrow g(2) = 29.$

$g'(x) = 12x^2 - 6x + 5 \Longrightarrow g'(2) = 41.$

$g''(x) = 24x - 6 \Longrightarrow g''(2) = 42.$

$$g'''(x) = 24 \Longrightarrow g'''(2) = 24.$$

$$g^{(m)}(x) = 0, \text{ for all } m \geq 4 \Longrightarrow g^{(m)}(2) = 0.$$

Then $g(x) = \sum_{k=0}^{\infty} \frac{g^{(k)}(2)}{k!}(x-2)^k$

$$= g(2) + g'(2)(x-2) + \frac{g''(2)}{2!}(x-2)^2 + \frac{g'''(2)}{3!}(x-2)^3$$

$$= 29 + 41(x-2) + \frac{42}{2!}(x-2)^2 + \frac{24}{3!}(x-2)^3$$

$$= 29 + 41(x-2) + 21(x-2)^2 + 4(x-2)^3.$$

---

(2) $g(x) = x^2 \ln x$, at $a = 1$. Exc.

Ex. Expand the func. $g(x)$ as indicated.

(1) $g(x) = e^x$ about $a = 1$.

Soln. $e^x = e^{(x-1)+1} = e\,e^{x-1} = e \sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$, $x \in \mathbb{R}$.

(2) $g(x) = \frac{1}{x}$ about $a = 3$.

Soln. $\frac{1}{x} = \frac{1}{(x-3)+3} = \frac{1}{3\left[\frac{x-3}{3} + 1\right]} = \frac{1}{3}\sum_{k=0}^{\infty} (-1)^k \left(\frac{x-3}{3}\right)^k$

Then $\frac{1}{x} = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^{k+1}}(x-3)^k$.

Note that this series converges for $\left|\frac{x-3}{3}\right| < 1$, that is, for $|x-3| < 3$, i.e. $x \in (0, 6)$.

---

(3) $g(x) = \sin x$ about $a = -\pi/2$.

Soln. $\sin x = \sin\left(\left(x+\frac{\pi}{2}\right) - \frac{\pi}{2}\right)$

$$= \sin\left(x+\frac{\pi}{2}\right)\underbrace{\cancel{\cos\left(\frac{\pi}{2}\right)}}_{0} - \sin\left(\frac{\pi}{2}\right)\cos\left(x+\frac{\pi}{2}\right)$$

$$= -\cos\left(x+\frac{\pi}{2}\right)$$

$$= \cancel{-}\sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{(2k)!}\left(x+\frac{\pi}{2}\right)^{2k}, \quad x \in \mathbb{R}.$$

---

(4) $g(x) = \ln x$, about $a = 1$.

Soln. $\ln x = \ln(1 + (x-1))$

$$= \sum_{k=1}^{\infty} \frac{(-1)^k}{k}(x-1)^k.$$

Recall that $\ln(1+t) = \sum_{k=1}^{\infty} \frac{(-1)^k}{k} t^k$ which conv. for $|t| < 1$.

Note that this series conv. for $|x-1| < 1$, that is, for $0 < x < 2$.

---

(5) $g(x) = \ln x$, about $a = e$.

Soln. $\ln x = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} (x-1)^k$.

$$\boxed{\frac{x}{e} \longrightarrow x} \Longrightarrow \ln\left(\frac{x}{e}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \left(\frac{x}{e} - 1\right)^k$$

$$= \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} \left(\frac{x-e}{e}\right)^k.$$

Then $\ln x - 1 = \ln x - \ln e = \ln\left(\frac{x}{e}\right) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}(x-e)^k}{k\, e^k}$.

Thus, $\ln x = 1 + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\, e^k} (x-e)^k$.

Note that this series converges for $\left|\frac{x-e}{e}\right| < 1$, that is, for $|x-e| < 2$. So, $x \in (0, 2e)$.

In general, we have the following fact.

Fact: $\boxed{\ln x = \ln a + \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k\, a^k} (x-a)^k, \quad a > 0.}$

This series converges for $x \in (0, 2e)$.

Ex. (Multiplication of power series).

Find the first 3 nonzero terms in the Maclaurin series for $f(x) = e^x \sin x$.

Soln. $e^x \sin x = \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots\right)\left(x - \frac{x^3}{3!} + \cdots\right)$

$= x + x^2 + \frac{1}{3}x^3 + \cdots .$

Ex. (Division of power series).

Find the first 3 nonzero terms in the Maclaurin series for $f(x) = \tan x$.

Soln. $\tan x = \frac{\sin x}{\cos x} = \dfrac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots}{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots}$

$\overset{\text{Exc.}}{=} x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$

This lecture: Taylor series.

Next lecture: Polar coordinates.

Searching keywords:

- Test the series for convergence or divergence افحص المتتالية للتقارب أو التباعد
- Taylor series, Taylor’s theorem, Maclaurin series, power series
- The University of Jordan الجامعة الأردنية
- Calculus II 2 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/102.aspx

For any comments or concerns, please use my email to contact me.

د. بهاء محمود الزالق
The University of Jordan
Dr. Baha Alzalg
baha2math@gmail.com