Series Convergence Tests

Q: How can we test the convergence of an infinite series $\sum_{k=1}^{\infty} a_k$?

Convergence tests are methods of testing for the convergence (conv.) or divergence (div.) of a given series.

1) The divergence test (D.T.) (or the k^{th}-term test).

For $\sum_{k=1}^{\infty} a_k$, we have the following:

(A) If $\lim_{k\to\infty} a_k \neq 0$ or DNE, then $\sum_{k=1}^{\infty} a_k$ is div.

(B) If $\sum_{k=1}^{\infty} a_k$ is conv., then $\lim_{k\to\infty} a_k = 0$.

Ex. Test for convergence or divergence.

(1) $\sum_{k=1}^{\infty} \frac{k}{k+2}$.

Soln. $\lim_{k\to\infty} \frac{k}{k+2} = 1 \neq 0 \xRightarrow{\text{D.T.}} \sum_{k=1}^{\infty} \frac{k}{k+2}$ is div.

(2) $\sum_{k=3}^{\infty}\left(1+\frac{2}{k}\right)^k$.

Soln. $\lim_{k\to\infty}\left(1+\frac{2}{k}\right)^k = e^2 \neq 0 \xRightarrow{\boxed{D.T.}} \sum_{k=3}^{\infty}\left(1+\frac{2}{k}\right)^k$ is div.

(3) $\sum_{k=5}^{\infty}\frac{1}{2+3^{-k}}$.

Soln.

$\lim_{k\to\infty}\frac{1}{2+3^{-k}} = \frac{1}{2+0} = \frac{1}{2}\ (\neq 0) \xRightarrow{\boxed{D.T.}} \sum_{k=5}^{\infty}\frac{1}{2+3^{-k}}$ is div.

Recall that

Let $r\in\mathbb{R}$, then $\lim_{k\to\infty} r^k = \begin{cases} 0 & \text{, if } |r|<1, \\ 1 & \text{, if } r=1, \\ \text{DNE} & \text{, if } r\le -1 \text{ or } r>1. \end{cases}$

and $\lim_{k\to\infty} r^{-k} = \begin{cases} 0 & \text{, if } r\le -1 \text{ or } r>1, \\ 1 & \text{, if } r=1, \\ \text{DNE} & \text{, if } |r|<1. \end{cases}$

(4) $\sum_{k=1}^{\infty}(-1)^k\frac{3k}{k+1}$.

Soln. $(-1)^k\frac{3k}{k+1} = \begin{cases} \frac{3k}{k+1} \longrightarrow 3 & \text{if } k \text{ is even.} \\ \frac{-3k}{k+1} \longrightarrow -3 & \text{if } k \text{ is odd.} \end{cases}$

Thus $\lim_{k\to\infty}(-1)^k\frac{3k}{k+1} \neq 0$ (DNE).

By the D.T., $\sum_{k=1}^{\infty}(-1)^k\frac{3k}{k+1}$ is div.

(5) $\sum_{k=1}^{\infty} k^2 \sin\left(\frac{1}{k}\right)$.

Soln. $\lim_{k\to\infty} k^2 \sin\left(\frac{1}{k}\right) = \lim_{k\to\infty} \frac{\sin\left(\frac{1}{k}\right)}{\frac{1}{k^2}} \quad \left(\frac{0}{0}\right)$

$$\overset{\text{L.R.}}{=} \lim_{k\to\infty} \frac{\cancel{-}\frac{1}{\cancel{k^2}} \cos\left(\frac{1}{k}\right)}{\cancel{-}\frac{2}{\cancel{k^3}}}$$

$$= \lim_{k\to\infty} \frac{k}{2} \cos\left(\frac{1}{k}\right)$$

$$= \infty \ (\neq \text{zero}).$$

Thus, $\sum_{k=1}^{\infty} k^2 \sin\left(\frac{1}{k}\right)$ is a div. series.

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(6) $\sum_{k=3}^{\infty} \left(3k - \ln\left(4e^{3k}+5\right)\right)$.

Soln. $\lim_{k\to\infty} \left(3k - \ln\left(4e^{3k}+5\right)\right) = \lim_{k\to\infty} \left(\ln e^{3k} - \ln\left(4e^{3k}+5\right)\right)$

$$= \lim_{k\to\infty} \ln\left(\frac{e^{3k}}{4e^{3k}+5}\right)$$

($y = \ln x$ is cts at $x = \frac{1}{4}$)

$$= \ln\left(\lim_{k\to\infty} \frac{e^{3k}}{4e^{3k}+5}\right)$$

$$\overset{\text{L.R.}}{=} \ln\left(\lim_{k\to\infty} \frac{\cancel{3}\cancel{e^{3k}}}{\underset{4}{\cancel{12}}\, \cancel{e^{3k}}}\right)$$

$$= \ln\left(\frac{1}{4}\right) \neq 0$$

Thus, $\sum_{k=3}^{\infty} \left(3k - \ln\left(4e^{3k}+5\right)\right)$ is div.

Note: If $\lim_{k\to\infty} a_k = 0$, we cannot conclude that $\sum a_k$ is conv.

Ex. Note that $\lim_{k\to\infty} \frac{1}{k} = 0$.

$\sum \frac{1}{k}$ is called the harmonic series.

Below we see that this series is div.

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② The P-test (P.T.)

The p-series $\sum_{k=1}^{\infty} \frac{1}{k^p}$ is

conv. if $p > 1$ div. if $p \leq 1$.

Ex. Test for convergence or divergence.

(1) The harmonic series $\sum \frac{1}{k}$ is div. $(p=1)$.

(2) $\sum_{k=1}^{\infty}\left(\frac{1}{4^k} - \frac{1}{k^4}\right) = \sum_{k=1}^{\infty}\left(\frac{1}{4}\right)^k - \sum_{k=1}^{\infty}\frac{1}{k^4}$

conv. $(|\frac{1}{4}|<1)$ conv. $(p=4>1)$

conv. + conv. $\Rightarrow$ conv.

③ The integral test (I.T.)

For $\sum_{k=0}^{\infty} a_k$, let $f(k) = a_k$.

If f is a continuous, positive, decreasing func. on $[1,\infty)$, then $\sum_{k=1}^{\infty} a_k$ and $\int_1^{\infty} f(x)\,dx$ are either both series and integral conv. or both div.

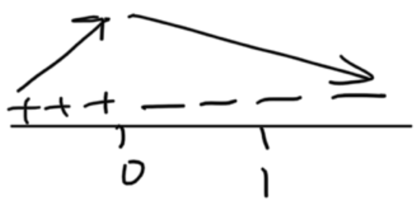
Ex. Test for convergence or divergence.

(i) $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$.

Soln. $f(x) = \frac{1}{x^2+1}$, $x \geq 1$.

f is cts and $f \geq 0$.

$f'(x) = \frac{-2x}{(x^2+1)^2} = 0$ when $x = 0$



then f is decreasing on $[1,\infty)$.

$$\int_1^{\infty} \frac{dx}{x^2+1} = \lim_{n\to\infty} \int_1^{n} \frac{dx}{x^2+1} = \lim_{n\to\infty} \tan^{-1}x \Big|_1^{n}$$

$$= \lim_{n\to\infty} \left(\tan^{-1}(n) - \tan^{-1}(1)\right)$$

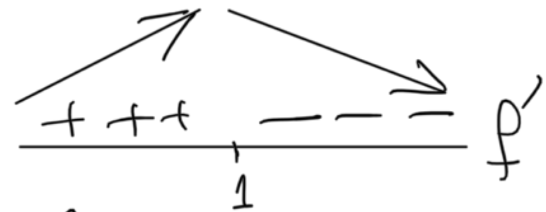
$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \quad \text{Conv.}$$

$\int_1^{\infty} \frac{dx}{x^2+1}$ conv. $\xRightarrow{\boxed{\text{I.T.}}}$ $\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ is conv.

(2) $\sum_{k=1}^{\infty} k e^{-k}$.

Soln. $f(x) = x e^{-x}$, $x \geq 1$.

f is cts, $f \geq 0$, and



$f'(x) = -x e^{-x} + e^{-x} = (1-x) e^{-x} = 0$ when $x = 1$

hence f is decreasing on $[1, \infty)$.

$$\int_1^{\infty} x e^{-x} dx = \lim_{n \to \infty} \int_1^n x e^{-x} dx$$

dv	u	
e^{-x}	x	$+$
$-e^{-x}$	1	$-$
e^{-x}	0	$+$

$$= \lim_{n \to \infty} \left(-x e^{-x} - e^{-x}\right]_1^n$$

$$= \lim_{n \to \infty} -e^{-x}(x+1)\Big|_1^n$$

$$= 2e^{-1} \quad \text{Conv.}$$

$\int_1^{\infty} x e^{-x} dx$ is conv. $\xRightarrow{\boxed{I.T.}}$ $\sum_{k=1}^{\infty} k e^{-k}$ is conv.

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(3) $\sum_{k=1}^{\infty} k^2 e^{-k^3}$.

Soln. $f(x) = x^2 e^{-x^3}$, $x \geq 1$.

f is cts and $f \geq 0$.

$$f'(x) = (x^2)(-3x^2) e^{-x^2} + (2x) e^{-x^3}$$

$$= x e^{x^3}(-3x^2 + 2)$$

$f'(x) = 0$ when $x = 0, \sqrt[3]{\frac{2}{3}}$

$$\begin{array}{ccccccc} & --- & 0 & ++++ & \sqrt[3]{\frac{2}{3}} & ---- & 1 \end{array}$$

So, $f(x)$ is decreasing on $[1, \infty)$.

$$\int_1^\infty x^2 e^{-x^3}\,dx = \lim_{n\to\infty} \int_1^n x^2 e^{-x^3}\,dx$$

let $u = x^3$
then $du = 3x^2 dx$
$x = 1 \to u = 1$
$x = n \to u = n^3$

$$= \lim_{n\to\infty} \int_1^{n^3} \cancel{x^2} e^{-u} \frac{du}{3\cancel{x^2}}$$

$$= \frac{1}{3} \lim_{n\to\infty} \int_1^{n^3} e^{-u}\,du$$

$$= \frac{1}{3} \lim_{n\to\infty} -e^{-u} \Big|_1^{n^3}$$

$$= \frac{1}{3} \lim_{n\to\infty} \left(-e^{-n^3} + e^{-1}\right)$$

$$= \frac{1}{3e} \;;\; \underline{\text{conv.}}$$

By I.T., $\sum_{k=1}^{\infty} k^2 e^{-k^3}$ is also conv.

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(4) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$. Exc.

(5) $\sum_{k=1}^{\infty} \frac{\sqrt{1+\frac{1}{k}}}{k^2}$. Exc.

(6) $\sum_{k=1}^{\infty} k e^{k}$. Exc.

(7) $\sum_{k=1}^{\infty} \frac{\tan^{-1} k}{1+k^2}$. Exc.

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④ The comparison test (C.T.).

If $0 < a_k \le b_k$ for all k, then

(A) If $\sum b_k$ is conv., then $\sum a_k$ is also conv.

(B) If $\sum a_k$ is div., then $\sum b_k$ is also div.

Ex. Test for convergence or divergence.

(1) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$.

Soln. $k^2+1 \ge k^2$ which implies that $\dfrac{1}{k^2+1} \le \dfrac{1}{k^2}$

and $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$ is conv. (P-series ($p = 2 > 1$)).

By C.T., it follows that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+1}$ is conv.

(2) $\displaystyle\sum_{k=5}^{\infty} \frac{1}{k-2}$.

Soln. $k > k-2$, hence $\dfrac{1}{k} < \dfrac{1}{k-2}$.

$\displaystyle\sum_{k=5}^{\infty} \frac{1}{k}$ is div. (harmonic series P.T. with $p=1$) $\xRightarrow{\text{C.T.}}$ $\displaystyle\sum_{k=5}^{\infty} \frac{1}{k-2}$ is div.

(3) $\sum_{k=1}^{\infty} \frac{11}{k^3+5k}$ is conv. by C.T., because

$\frac{11}{k^3+5k} \leq \frac{11}{k^3}$ and $\sum_{k=1}^{\infty} \frac{11}{k^3}$ is conv. by P.T.

(4) $\sum_{k=1}^{\infty} \frac{5^{k+1}}{2^{k-1}}$ is div. by C.T., because

$\frac{5^{k+1}}{2^{k-1}} \geq \frac{5^k}{2^k} = \left(\frac{5}{2}\right)^k$ and $\sum_{k=1}^{\infty} \left(\frac{5}{2}\right)^k$ is div (G.S. $\frac{5}{2} > 1$).

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(5) $\sum_{k=1}^{\infty} \frac{k e^{-k^2}}{4+e^{-k}}$ is conv. by C.T., because

$\frac{k e^{-k^2}}{4+e^{-k}} \leq k e^{-k^2}$ and it can be shown

that by the I.T. that $\sum_{k=1}^{\infty} k e^{-k^2}$ is conv.

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(6) $\sum_{k=1}^{\infty} \frac{2+\cos k}{k}$

Soln. $-1 \leq \cos k \leq 1$, hence $\frac{1}{k} \leq \frac{2+\cos k}{k} \leq \frac{3}{k}$.

The harmonic series $\sum \frac{1}{k}$ is div. (by P.T.).

So, by C.T., the series $\sum_{k=1}^{\infty} \frac{2+\cos k}{k}$ is div.

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(7) $\sum_{k=1}^{\infty} \frac{k!}{(k+2)!}$ is conv. by C.T., because

$$\frac{k!}{(k+2)!} = \frac{\cancel{k!}^{\,1}}{(k+2)(k+1)\cancel{k!}} = \frac{1}{k^2+3k+2} \leq \frac{1}{k^2}$$

and $\sum \frac{1}{k^2}$ is conv. by P.T. $(p=2>1)$.

(8) $\sum_{k=1}^{\infty} \frac{4}{(2+4k)^2}$. Exc.

(9) $\sum_{k=3}^{\infty} \frac{5}{k^3+9k}$. Exc.

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⑤ The limit comparison test (L.C.T.).

For $a_k, b_k > 0$. If

$L = \lim_{k\to\infty} \frac{a_k}{b_k}$

- $L>0$ $(L \neq \infty)$, then either both $\sum a_k$ and $\sum b_k$ conv. or both div.
- $L=0$ and $\sum b_k$ conv., then $\sum a_k$ conv.
- $L=\infty$ and $\sum b_k$ conv., then $\sum a_k$ div.

Ex. Test for convergence and divergence.

(1) $\sum_{k=1}^{\infty} \frac{k^4+5k^2+1}{k^5+2k^4+3}$ a_k

$b_k = \dfrac{\text{الحد الذي يحمل أكبر أس في البسط}}{\text{الحد الذي يحمل أكبر أس في المقام}}$

Soln. $b_k = k^4/k^5 = 1/k$.

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{(k^4+5k^2+1)}{k^5+2k^4+3} \cdot \frac{1}{1/k} = \lim_{k\to\infty} \frac{k^5+5k^3+k}{k^5+2k^4+3k} = 1 > 0.$$

$\sum \frac{1}{k}$ is div. (P.T.) $\xRightarrow{\text{L.C.T.}}$ $\sum \frac{k^4+5k^2+1}{k^5+2k^4+3}$ is also div.

(2) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2-2k+7}{k^5+5k^4-3k^3+2k+1}$ ← a_k

Soln. $b_k = \frac{k^2}{k^5} = \frac{1}{k^3}$ and

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^5-2k^4+7k^3}{k^5+5k^4-3k^3+2k+1} = 1 > 0.$$

$\displaystyle\sum_{k=1}^{\infty} b_k = \sum_{k=1}^{\infty} \frac{1}{k^3}$ conv. (by P.T.) $\xRightarrow{\text{L.C.T.}}$ $\sum a_k$ conv.

(3) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+1)\sqrt{k} + k\sqrt{k+1}}$

Soln. $k^{3/2}$ + lower terms $\qquad$ $k^{3/2}$ + lower terms

let $b_k = \frac{1}{2k^{3/2}}$, then

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{2k^{3/2}}{(k+1)\sqrt{k} + k\sqrt{k+1}} = \frac{2}{2} = 1 > 0.$$

$\sum b_k = \frac{1}{2} \sum \frac{1}{k^{3/2}}$ conv. by P.T. ($p = 3/2 > 1$)

$\xRightarrow{\text{L.C.T.}}$ $\sum a_k$ is also conv.

(4) $\displaystyle\sum_{k=1}^{\infty} \frac{2k+1}{(k+1)\sqrt{k} + k^2\sqrt{k+1}}$

Soln. $k^{3/2}$ + lower terms $\qquad$ $k^{5/2}$ + lower terms

let $b_k = \dfrac{k}{k^{5/2}} = \dfrac{1}{k^{3/2}}$.

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{(2k+1)\,k^{3/2}}{(k+1)\sqrt{k} + k^2\sqrt{k+1}} = 2 > 0.$$

$\sum b_k = \sum 1/k^{3/2}$ conv. $\xRightarrow{\text{L.C.T.}}$ $\sum a_k$ conv.

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(5) $\displaystyle\sum_{k=1}^{\infty} \frac{\ln k}{k^{1.4}}$.

Soln. ~~We could take $b_k = \frac{k^p}{k^{1.4}}$ where $0<p<0.4$.~~

Take $b_k = \dfrac{1}{k^{1.2}}$, then $\sum b_k = \sum \dfrac{1}{k^{1.2}}$ conv. by P.T. $(p=1.2>1)$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^{1.2}\ln k}{k^{1.4}} = \lim_{k\to\infty} \frac{\ln k}{k^{0.2}}$$

$$\overset{\text{L.R.}}{=} \lim_{k\to\infty} \frac{1/k}{0.2\,k^{-0.8}} = \lim_{k\to\infty} \frac{1}{0.2\,k^{0.2}} = 0.$$

$\sum b_k$ conv. $\xRightarrow{\text{L.C.T.}}$ $\sum a_k$ conv.

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(6) $\displaystyle\sum_{k=1}^{\infty} \sin\left(\frac{\pi}{k}\right)$. $\leftarrow a_k$

<u>Soln.</u> Take $b_k = \frac{\pi}{k}$.

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{\sin\left(\frac{\pi}{k}\right)}{\frac{\pi}{k}} = \lim_{x\to 0} \frac{\sin x}{x} = 1.$$

$$\sum \frac{\pi}{k} \text{ div. } \xRightarrow{\text{L.C.T.}} \sum \sin\left(\frac{\pi}{k}\right) \text{ div.}$$

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(7) $\displaystyle\sum_{k=1}^{\infty} \frac{e^{1/k}+1}{k^3}$. <u>Exc.</u>

(8) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k!}\right)$. <u>Exc.</u>

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(6) <u>The ratio test (Ratio T.)</u>

For the series $\displaystyle\sum_{k=1}^{\infty} a_k$, if

$$\ell = \lim_{k\to\infty} \left(\frac{a_{k+1}}{a_k}\right)$$

- $\ell < 1$, then the series is conv.
- $\ell > 1$ (or ∞), then the series is div.
- $\ell = 1$, then the test is inconclusive.

The test is very useful with exponentials and factorials.

Ex: Test for convergence or divergence

(1) $\sum_{k=1}^{\infty} \frac{k}{2^k} \leftarrow a_k$ (let)

Soln. $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{k+1}{\cancel{2^{k+1}}_{2}} \cdot \frac{\cancel{2^k}}{k}$

$= \frac{1}{2} \lim_{k\to\infty} \frac{k+1}{k}$

$= \frac{1}{2} < 1.$

$\xrightarrow{\text{Ratio T.}} \sum a_k$ converges.

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(2) $\sum_{k=1}^{\infty} \frac{k!}{e^k} \leftarrow a_k$ (let)

Soln. $\lim_{k\to\infty} \frac{(k+1)!}{e^{k+1}} \cdot \frac{e^k}{k!} = \lim_{k\to\infty} \frac{(k+1)\cancel{k!}}{e \cdot \cancel{e^k}} \cdot \frac{\cancel{e^k}}{\cancel{k!}}$

$= \lim_{k\to\infty} \frac{k+1}{e} = \infty.$

$\xrightarrow{\text{Ratio T.}} \sum a_k$ diverges.

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(3) $\sum_{k=1}^{\infty} \frac{2^k}{k!} \leftarrow a_k$

Soln. $\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{\cancel{2^{k+1}}^{2}}{\cancel{(k+1)!}_{k+1}} \cdot \frac{\cancel{k!}}{\cancel{2^k}} = \lim_{k\to\infty} \frac{2}{k+1} = 0 < 1.$

$\xrightarrow{\text{Ratio T.}}$ the series $\sum a_k$ is convergent.

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(4) $\displaystyle\sum_{k=1}^{\infty} \frac{k^k}{k!} \leftarrow a_k$.

Sol. $\displaystyle\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(k+1)^{k+1}}{(k+1)!} \cdot \frac{k!}{k^k}$

$$= \lim_{k\to\infty} \frac{\cancel{(k+1)}\,(k+1)^k}{\cancel{(k+1)}\,\cancel{k!}} \quad \frac{\cancel{k!}}{k^k}$$

$$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^k$$

$$= \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^k$$

$$= e > 1 .$$

$\xrightarrow{\text{Ratio T.}}$ $\sum a_k$ is a divergent series.

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7) The root test (Root T.)

For the series $\displaystyle\sum_{k=1}^{\infty} a_k$, if

$\displaystyle l = \lim_{k\to\infty} \sqrt[k]{a_k}$

- $\to l < 1$, then the series is conv.
- $\to l > 1$ (or ∞), then the series is div.
- $\to l = 1$, then the test is inconclusive

The test is very useful with exponentials.

Ex. Test for convergence or divergence.

(1) $\sum_{k=1}^{\infty} k^{50} e^{-k} \leftarrow a_k$

Soln. $\lim_{k\to\infty} (a_k)^{\frac{1}{k}} = \lim_{k\to\infty} \left(k^{50} e^{-k}\right)^{\frac{1}{k}}$

$= \lim_{k\to\infty} k^{50/k} e^{-1}$

$= e^{-1} \left(\lim_{k\to\infty} k^{1/k}\right)^{50}$

$= e^{-1} (1)^{50}$

$= 1/e$.

Recall $\lim_{k\to\infty} k^{\frac{1}{k}} = 1$

Root T. $\Rightarrow \sum a_k$ is conv.

(2) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$.

Soln. $\lim_{k\to\infty} \left[\left(\frac{k}{k+1}\right)^{k^{\cancel{2}}}\right]^{\frac{1}{\cancel{k}}} = \lim_{k\to\infty} \left(\frac{k}{k+1}\right)^{k}$

$= \lim_{k\to\infty} \left[\left(\frac{k+1}{k}\right)^{k}\right]^{-1}$

$= \left[\lim_{k\to\infty} \left(1+\frac{1}{k}\right)^{k}\right]^{-1}$

$= e^{-1} < 1$.

$\boxed{\text{Root T.}} \Rightarrow \sum \left(\frac{k}{k+1}\right)^{k^2}$ Conv.

(3) $\displaystyle\sum_{k=3}^{\infty} \left(\frac{5k+1}{6k-7}\right)^{k}$ is conv. by Root T., because

$$\lim_{k\to\infty} \left[\left(\frac{5k+1}{6k-7}\right)^{\cancel{k}}\right]^{\frac{1}{\cancel{k}}} = \lim_{k\to\infty} \frac{5k+1}{6k-7} = \frac{5}{6} < 1.$$

(4) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k}{k^2 4^k}$ is conv. by Root T., because

$$\lim_{k\to\infty} \left(\frac{3^{\cancel{k}}}{k^{2/k} 4^{\cancel{k}}}\right)^{\frac{1}{\cancel{k}}} = \lim_{k\to\infty} \frac{3}{k^{2/k}\, 4}$$

$$= \frac{3}{4} \frac{1}{\left(\lim_{k\to\infty} k^{1/k}\right)^2}$$

$$= \frac{3}{4} \frac{1}{1^2} = \frac{3}{4} < 1.$$

This lecture: Series convergence tests.
Next lecture: Alternating series.

Searching keywords:

- Test the series for convergence or divergence افحص المتتالية للتقارب أو التباعد
- Divergence test, kth-term test
- Integral test, comparison test, limit comparison test
- Ratio test, root test
- The University of Jordan الجامعة الأردنية
- Calculus II 2 تفاضل وتكامل
- Baha Alzalg بهاء الزالق

References: See the course website

http://sites.ju.edu.jo/sites/Alzalg/Pages/102.aspx

For any comments or concerns, please use my email to contact me.

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