The University of Jordan		**Department of Mathematics**
Student's Name:..................		**Instructor's Nume:**..................
Student's Number:...............		**Class Time:**..........................

Second Exam ⋄ Calculus I (0301101) ⋄ Fall 2016
Note: This exam is composed of 15 questions. You have 60 minutes to finish

For questions 1-12, fill in the blank with the correct answer. Only correct answer count. [1.5 point each].

1. If $4x^2 + 2xy + y^2 = 12$, then $\frac{dy}{dx}$ at the point $(1, 2)$ is equal to -2.
2. $\lim_{h\to 0} \dfrac{8(1+h)^8 - 8}{h} = 64$.
3. The equation of the tangent line to the graph of $y = \sin^{-1}\left(\frac{x}{y}\right)$ at the origin $\boldsymbol{y} = \frac{x}{2}$.
4. $\dfrac{d}{dx}\left[\log_3(x^2 + e^2)\right] = \dfrac{2x}{(x^2 + e^2)\ln 3}$.
5. $f(x) = \dfrac{\sin x}{\sqrt{4 - x^2}}$ is continuous on $(-2, 2)$.
6. $\lim_{x\to\frac{\pi}{2}} e^{\sec(x)} = 0$.
7. If $g(0) = 2, g'(0) = 3$ and $f(x) = \dfrac{4 - 3e^{2x}}{x + g(x)}$, then $f'(0) = -4$.
8. $\lim_{x\to 0} \dfrac{1 - \cos^2(4x)}{x^2} = 16$.
9. If $f(x) = \sec^3(7^x)$, then $f'(x) = 3\ln(7)\, 7^x \sec^3(7^x)\tan(7^x)$.
10. If $f(x) = \ln(x + 8)$, then $\lim_{x\to 2} \dfrac{f(x) - f(2)}{x - 2} = \dfrac{1}{10}$.
11. The vertical asymptote of $f(x) = \dfrac{x - 2}{x^2 - 5x + 6}$ is $x = 3$.
12. $\lim_{x\to -2} \dfrac{2x + 4}{|x| + 2} = 0$.

For questions 13-15, sufficient work must be shown to receive credit.

13. [4 points] Find $\lim_{x\to-\infty}\left(\sqrt{x^2+3x}+x\right)$.
Solution:

$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{x^2+3x}+x\right) &= \lim_{x\to-\infty}\left(\sqrt{x^2+3x}+x\right)\frac{\left(\sqrt{x^2+3x}-x\right)}{\left(\sqrt{x^2+3x}-x\right)} \\
&= \lim_{x\to-\infty}\frac{x^2+3x-x^2}{\sqrt{x^2+3x}-x} = \lim_{x\to-\infty}\frac{3x}{\sqrt{x^2+3x}-x} \\
&= \lim_{x\to-\infty}\frac{3x}{-x\sqrt{1+\frac{3}{x}}-x} = \lim_{x\to-\infty}\frac{3}{-\sqrt{1+\frac{3}{x}}-1} \\
&= -\frac{3}{2}.
\end{aligned}$$

14. [4 points] Use linear approximation to estimate $\sqrt[3]{1001}$.
Solution:

Let $f(x)=\sqrt[3]{x+1001}$, then $f'(x)=\dfrac{1}{3\sqrt[3]{(x+1001)^2}}$,
$f(0)=10, f'(0)=\frac{1}{300}$. So, $L(x)=10+\frac{x}{300}$, and hence $\sqrt[3]{1001}\approx L(1)=10+\frac{1}{300}$.

15. [4 points] Differentiate $f(x)=\dfrac{\sqrt[4]{x}\,\sin^9(x)}{(x-1)^7e^{x^2}}$.

Solution: Note that

$$\begin{aligned}
\ln(f(x)) &= \ln\left(\frac{\sqrt[4]{x}\,\sin^9(x)}{(x-1)^7e^{x^2}}\right) \\
&= \ln\left(\sqrt[4]{x}\right)+\ln\left(\sin^9(x)\right)-\ln\left((x-1)^7\right)-\ln\left(e^{x^2}\right) \\
&= \tfrac{1}{4}\ln(x)+9\ln(\sin(x))-7\ln(x-1)-x^2.
\end{aligned}$$

Then

$$\frac{f'(x)}{f(x)} = \frac{1}{4x}+9\cot x-\frac{7}{x-1}-2x.$$

It follows that

$$\begin{aligned}
f'(x) &= f(x)\left(\frac{1}{4x}+9\cot x-\frac{7}{x-1}-2x\right) \\
&= \frac{\sqrt[4]{x}\,\sin^9(x)}{(x-1)^7e^{x^2}}\left(\frac{1}{4x}+9\cot x-\frac{7}{x-1}-2x\right).
\end{aligned}$$